



Liner Service Network Design

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Ph.D. thesis

Liner Service Network Design

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Preface

This PhD study is part of the ENERPLAN project funded by *The Danish Council for Strategic Research*. ENERPLAN is an abbreviation of *Energy Efficient Transport Planning* and aims at developing green, logistic decision support tools to assist in reducing energy consumption, thereby decreasing the impact of container shipping on the environment. ENERPLAN is a collaboration between Maersk Line, DTU Management Engineering and the IT University of Copenhagen (ITU).

The PhD study was performed at DTU Management Engineering, Technical University of Denmark, from October 2009 to October 2012. I have been privileged to work with Maersk Line and their optimization group, who have readily assisted me with data and domain knowledge of the liner shipping business. Professor David Pisinger supervised the study, Ph.D. and Optimization Manager at Maersk Line Mikkel M. Sigurd acted as a second supervisor located at Maersk Line. Associate Professor Rune Møller Jensen acted as supervisor at the IT University of Copenhagen.

The thesis consist of two overviews (Chapter 1 and 7), five research papers (Chapter 2, 3, 4, 5 and 6) and a conclusion (Chapter 8). The research papers are all co-authored. Each research paper is relatively self-contained and has an individual bibliography.

The PhD thesis is divided into four parts. The first part contains background for the project and an introduction to the liner shipping network design problem. The second part of the thesis concerns tangible operational problems related to network design. The third part is concerned with mathematical modelling of the liner shipping network design problem and the final part contains a conclusion and directions for future work.

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I would like to thank my colleagues at DTU Management Engineering as well as the optimization group at Maersk Line for fruitful discussions, interest in my project and a pleasant work

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Abstract

This thesis concerns design of liner shipping networks using operations research to optimize liner shipping networks at the strategic, tactical and operational level. Liner shipping networks are often compared to public transit networks as they consist of a set of scheduled sailings connecting at main ports to consolidate freight on large vessels. The liner shipping network is constrained by the composition and size of the vessel fleet and the objective is to maximise the profit of freighting containers between origins and destinations by a high utilization of the capacity of the fleet.

The research field of liner shipping network design is relatively young and many open research questions exists. Among others, a unified and rich mathematical model formulating the main characteristics of the business domain has not been clearly described and exact methods for such mathematical models are still not able to solve significant instances of this complex optimization problem.

In this thesis two research directions are explored within the field: The first research direction contributes to basic research on the liner shipping network design problem by describing the domain seen from the perspective of a global liner shipping carrier, present public realistic data instances of the problem to perform computational experiments, and discuss alternative mathematical models of the problem. A domain description, literature survey and a base integer model of the problem is presented. It is accompanied by the description of a public benchmark suite of data instances and a heuristic column generation algorithm to present the first results for this benchmark suite.

The thesis contains a discussion of the complexity of the liner shipping network design problem in relation to mathematical models. The severe complexity of mathematical models of liner shipping network design is discussed in relation to research on the related classical multicommodity capacitated network design (MCND) and Pick-up-and-Delivery-Vehicle-Routing-Problem (PDVRP). It contains a discussion of applying state-of-the-art exact algorithms using Dantzig-Wolfe decomposition and presents ideas on the design of Branch-and-Cut/Branch-and-Price algorithms.

The second research direction is to identify and analyse tractable problems at the tactical and operational level of liner shipping network design. A mathematical model for revenue management of cargo flows considering the significant repositioning of empty containers (*Cargo Allocation Problem with Empty Repositioning*) is presented. The model is a multicommodity flow problem with inter-balancing constraints to account for empty repositioning. An arc-flow formulation is decomposed using the Dantzig-Wolfe principle. A linear relaxation of the path flow model is solved using delayed column generation applying a shortest path algorithm to solve the pricing problem. A feasible integer solution is calculated by rounding the fractional solution and adjusting flow balance constraints with leased containers. Computational results are reported for seven instances based on real-life shipping networks. Solving the relaxed linear path flow model with a column generation algorithm outperforms solving the relaxed linear arc flow model with the *CPLEX* barrier solver even for very small instances. The proposed algorithm is able to solve instances with 234 ports, 16278 demands over 9 time periods in 34 minutes. The integer solutions found by rounding down are computed in less than 5 seconds and the gap is within 0.01% from the upper bound of the linear relaxation.

At the operational level disruption management is of great concern to liner shippers as 70-80% of vessel round trips experience delays in at least one port. A novel mathematical model for handling a disruption using a series of recovery techniques is presented as the *The Vessel Schedule*

Recovery Problem. The model has been applied to four real life cases from Maersk Line, where the solutions found using the MIP model are comparable or superior to the realised recovery actions performed at Maersk Line. The cases are solved using a commercial solver *CPLEX*, and solutions are obtained within 10 seconds.

Lastly, we present a matheuristic, which may serve both research directions. The matheuristic aims at solving the liner shipping network design problem in its entirety using a construction heuristic. The heuristic aims at designing a neighbourhood and a local search method, which scales well to realistic sized networks. The heuristic is based upon improving the constructed solution by applying an IP model as a large scale neighbourhood to each service in the network. The IP is based on estimating the benefit of inserting and removing port calls within a predefined neighborhood of candidate ports. Furthermore, the heuristic applies delayed column generation to evaluate the resulting network using a warm start method denoted *delta column generation*. The IP neighborhood as well as the delta column generation scales well to larger instances and may hence be an inspiration to future method development, but the quality of the solutions may be improved upon by combining it with more advanced local search methods to remove and introduce services in the network. The matheuristic may also be applied at a tactical level, as an existing network may be used as input and improved upon by the heuristic. It is also possible to optimize upon a predefined subset of services, while maintaining a full view of the cargo flows in the network. This could prove a valuable decisions support tool for planners when the network is incrementally adjusted to new market situations and to evaluate the consequence of strategic decisions.

The thesis contributes to the understanding of the domain for the Liner Shipping Network Design Problem. Three different mathematical models for the problem are presented. Two heuristics to solve the first model has been implemented and computational results have been presented for small, medium and large scale instances. The thesis explores two planning problems related to liner shipping network design. The Cargo Allocation Problem with Empty Repositioning is the first model to simultaneously consider cargo allocation considering both ordinary cargo and empty repositioning of containers. Computational results confirm tractability of solving networks of realistic size in reasonable time with this model. We also present a novel model to handle disruptions in liner shipping, The Vessel Schedule Recovery Problem. Computational results for four common disruption scenarios from Maersk Line show comparable or improved results compared to the realised recovery actions.

Resumé

Denne afhandling omhandler design og optimering af liner shipping netværk for container skibe ved anvendelse af operations analyse. Et liner shipping netværk er et rutenetværk, der sejles efter en fastlagt sejlplan. Liner shipping netværk sammenlignes ofte med offentlige transport netværk såsom busser, metro og tog. Netværket er begrænset af det tilgængelige antal skibe og det forsøges at optimere på netværkets ruter som helhed for at optimere kapacitets udnyttelsen på skibene samtidig med indtjeningen fra fragt af containere. Der er relativt få studier af liner shipping netværk indenfor operations analyse og problemet er særdeles kompleks set i forhold til andre transport optimerings problemer.

Forskningsfeltet inden for liner shipping netværk design er relativt ubeskrevet og der er til stadighed mange åbne spørgsmål. Blandt andet beskrivelsen af en samlet rig matematisk model, der formulerer de vigtigste karakteristika for domænet. På nuværende tidspunkt er optimale løsnings metoder ikke i stand til at løse større instanser af dette komplekse optimerings problem.

I denne afhandling udforskes to forskningsretninger: Den første forsknings retning bidrager til grundforskning indenfor liner shipping netværk design problemet ved at beskrive domænet set fra et globalt rederis perspektiv, præsentere offentlige realistiske data instanser til at udføre beregningsmæssige eksperimenter og diskutere alternative matematiske modeller af problemet. En domæne beskrivelse, litteraturoversigt og en heltals model af problemet præsenteres. Det er ledsaget af en beskrivelse af en offentlig benchmark suite af data instanser og en heuristisk kolonne genererings algoritme, der præsenterer de første resultater for benchmark suiten.

Afhandlingen indeholder en diskussion af kompleksiteten af liner shipping netværk design problemet set i relation til matematisk modeller. De matematiske modeller for liner shipping netværk design relateres til forskningen indenfor klassiske Multicommodity Capacitated Network Design (MCND) og Pick-up-and-Delivery-Vehicle-Routing-Problem (PDVRP). Den indeholder en diskussion af anvendelsen af eksakte algoritmer ved hjælp af Dantzig-Wolfe dekomposition og design af Branch-and-Cut/Branch-and-Price algoritmer.

Den anden forsknings retning er at identificere og analysere håndterbare optimerings problemer på det taktiske og operationelle niveau af liner shipping netværk design. I afhandlingen præsenteres en matematisk model for revenue management af de transportede containere i netværket tager højde for den betydelige repositionering af tomme containere (*Cargo Allocation Problem with Empty Repositioning*). Modellen er en multicommodity flow formulering med balancerings begrænsninger, der tager højde for repositionering af tomme containere. En arc-flow formulering dekomponeres ved hjælp af Dantzig-Wolfe princippet til en path-flow formulering. En lineær relaxering af path-flow modellen løses med kolonne generering og en korteste vej algoritme til at løse pricing problemet. En heltals løsning beregnes ved at afrunde den fraktionelle løsning og justere for de tomme containere. Der rapporteres eksperimenter for syv instanser baseret på realistiske liner shipping netværk. Den relaxerede path-flow formulering løst med kolonne generering er væsentligt hurtigere end at løse den relaxerede arc-flow model med *CPLEX* barrier Solver selv for de mindre data instanser. Kolonne genererings algoritmen er i stand til at løse instanser med 234 havne, 16278 varer over 9 tidsperioder på 34 minutter. Heltallige løsninger baseret på afrunding beregnes på mindre end 5 sekunder og forskellen er under 0,01% fra den øvre grænse for den lineære relaxering.

På det operationelle niveau er håndtering af forsinkelser af stor bekymring for rederier, idet 70 - 80 % af alle services oplever forsinkelser i mindst én havn. En ny matematisk model for håndtering

af forsinkelser præsenteres som *The Vessel Schedule Recovery Problem*. Modellen er anvendt på fire cases fra Maersk Line, hvor de løsninger, der findes ved hjælp af MIP-modellen er sammenlignelige eller bedre end de realiserede løsninger implementeret af Maersk Line. Instanserne løses ved hjælp af en kommerciel løser *CPLEX*, og løsningerne findes indenfor 10 sekunder.

Endelig præsenterer vi en matheuristik, som understøtter begge forsknings retninger. Matheuristikken har til formål at løse liner shipping netværk design problemet i sin helhed ved hjælp af en konstruktions heuristik. Heuristikken har til formål at designe et nabolag og en lokal søgning, som skalerer godt til realistiske og mellemstore netværk. Heuristikken er baseret på forbedring af en løsning ved anvendelse af et large scale nabolag med en IP model for hver service i netværket. IP modellen er baseret på en estimeret gevinst for indsættelse og fjernelse af havnekald inden for et fastlagt nabolag af kandidat havne. Endvidere anvender heuristikken forsinket kolonne generering til at vurdere det resulterende netværk ved hjælp af en *warm start* metode kaldet *delta kolonne generering*. IP nabolaget såvel som delta kolonne genereringen skalerer godt til større instanser og kan derfor være en inspiration for design af fremtidige heuristikker, men kvaliteten af løsningerne kan forbedres ved at kombinere det med mere avancerede lokal søgningsmetoder til at justere antallet af services i netværket. Matheuristikken kan også anvendes på det taktiske niveau, hvor et eksisterende netværk anvendes som input og forbedres af heuristikken. Det er også muligt at optimere på en foruddefineret delmængde af services, hvor der regnes på det komplette netværks transport af containere. Dette kunne vise sig som et værdifuldt beslutnings støtte for planlæggere, når netværket trinvist justeres til nye markedsforhold eller til at vurdere konsekvensen af strategiske beslutninger.

Part I

Introduction

Chapter 1

Introduction and thesis motivation

Liner shipping is the backbone of world trade and many of our everyday products have at some point been transported on the global network of liner shipping companies. Liner shipping networks are comparable to urban transit networks freighting general cargo as opposed to passengers. As in urban transit networks the network is a composition of published schedules for ports of call, where cargo is consolidated onto larger vessels at central ports. The larger vessels connect a number of central ports not unlike urban metro systems via a designated service. At the central ports a number of services typically deploying smaller vessels distribute the cargo onto remaining ports similar to urban bus services. Seen from the perspective of operations research and mathematical models it can be modelled as a variant of the Multicommodity Capacitated Network Design (Gendron et al., 1999) with design balance requirements and cargo transshipments.

Liner shipping companies are an integrated part of the supply chains of international business and according to the World shipping council 60% of goods by value are transported by container ships (The World Shipping Council (2009)). This means that liner shipping network design affects not only the liner shipping companies, but the entire value chain from producer to consumer. Globalisation is part of the reason why liner shipping has been a growth industry for several decades in terms of the total volume carried by container ships. At the same time liner shipping is a driver of globalization because it provides integrated cost efficient supply chains. The increase in transportation of goods leads to increased CO_2 emissions and liner shipping contributes with 2,7 % of the worlds total CO_2 emission according to The World Shipping Council (2010).

The effect of increased emissions on the environment is of both political and social concern. The concept of *green logistics* as emphasized in Sbihi and Eglese (2010) state that optimizing capacity utilization and the total distance of existing transportation networks will in itself have a positive effect on the environment through reduced emissions. The liner shipping industry has great potential for applying the concept of *green logistics* because they are a large global transportation industry, where fuel consumption constitutes a significant portion of their total operational cost and their scale means that even marginal improvements in network efficiency may produce substantial reductions in fuel consumption. Using operations research to create cost efficient liner shipping networks with increased capacity utilization and the ability to reduce fuel consumption may help mitigate the negative effect of globalization on the environment. The impact of liner shipping on the global supply chain and the possibility to reduce global CO_2 emissions make the Liner Shipping Network Design Problem (hereafter LSNDP) an important optimization problem. This thesis is concerned with operations research within liner shipping aiming at designing a cost efficient global transportation network and minimizing CO_2 emissions of liner shipping through better transportation planning and increased utilization of capacity.

1.1 Motivation

This thesis is part of ENERPLAN (Energy Efficient Transport Planning). ENERPLAN is funded by *The Danish Council for Strategic Research* aiming at reducing the energy consumption of liner shipping through better transportation planning. The potential for improved efficiency of liner shipping networks is recognised by major stakeholders in the industry and the research communities, who have committed to research within areas related to LSNDP. The classification scheme for routing and scheduling problems within liner shipping by Kjeldsen (2011) counts 24 journal papers on the subject from the period 1969-2010, whereof 16 contributions are dated after the millennium. Since 2010 several references support an increased interest into various planning problems related to liner shipping network design e.g. Alvarez (2011); Brouer et al. (2011, 2012b); Jepsen et al. (2011); Kjeldsen (2011); Meng and Wang (2011, 2012); Wang and Meng (2011); Reinhardt and Pisinger (2011); Wang and Meng (2012a,b); Wang et al. (2013), but the research area is still young and many open research questions remain.

As a relatively young research area there is a need to explore the domain of liner shipping network design to a greater extend. Alternative descriptions of the exact domain of the liner shipping business are found in the literature with little input from key players in the industry. This makes it hard to agree upon a model and to distinguish between necessary and optional constraints. At the same time research in the area is concentrated to a few research groups and the field would benefit greatly from being able to attract more researchers with different expertise. As emphasized in the editorial of the INFOR special issue on Maritime Transportation by Christiansen and Fagerholt (2011), there is a need for public data sets for maritime shipping in general. In the case of LSNDP tailored benchmark instances are called for as the economic and geographic structure of liner shipping is very specific. The existence of public data sets will enable researchers to compare work and evaluate progress in mathematical model development and algorithmic design.

The LSNDP is a complex, large scale problem by nature challenging both models and methods within operations research, which means that modelling the LSNDP is an ongoing field of development. Network design problems in general are considered as hard optimization problems (Gendron et al., 1999) and the LSNDP is a special case of the capacitated network design problem requiring a consistent routing of the vessels constituting the capacitated network and charging a cost for transshipment between routes. At the same time the LSNDP has more degrees of freedom compared to capacitated network design problems as there is often no requirement to transport the demand, which is seen as a profit maximizing option. This means that among others the cut-set inequalities (Günlük, 1999), that are very successfully applied to capacitated network design, are not valid for most variants of the LSNDP. The LSNDP furthermore suffers from an objective function, where the majority of the cost is the fixed cost of deploying vessels/capacity on an arc (route generation for vessels), but instead of a cost of transportation per unit on open arcs, there is a revenue associated with the transport per unit of each commodity. This leads to highly fractional LP relaxations of the integer formulation resulting in poor lower bounds for Branch-and-Bound methods. The current literature on the LSNDP explore different variants of capacitated network design models in order to define rich, compact formulations of the LSNDP which is a challenging task. The search for a tight and descriptive formulation is ongoing.

The LSNDP is complex due to a high degree of freedom and the vast scale of real life problems. The literature presents various algorithms for different models of the LSNDP. In a brief overview of the field, some methods focus on a restricted solution space in terms of ports considered, an expert selection of services to optimize upon or restricted cargo routing (Shintani et al., 2007; Meng and Wang, 2011), while others solve to optimality small instances of different mathematical models of the LSNDP (Alvarez, 2009; Plum, 2010; Reinhardt and Pisinger, 2011). Heuristic methods for the liner shipping network design problem with no restrictions on the number of ports visited, the number of services or the routing of the cargo have been presented by Agarwal and Ergun (2008) and Alvarez (2009). A thorough review of these algorithms will be reviewed in Chapter 2.

The industry has no tradition for applying operations research in their planning process and hence demand has not driven research forward in the field of liner shipping as has been seen in

the field of tramp and industrial shipping. The industry has the leverage and size to benefit from the potential of decision support systems and luckily industry stakeholders are showing increased interest in this field of research to improve upon their network design process. The network design process is a complicated matter, which today consist of several sequential evaluations and iterative improvements to the network. Saving merely a fraction of operating costs or utilizing the capacity slightly better will produce huge cost savings and minimize carbon emission significantly simply due to the scale and size of this industry.

It is important to develop simple planning tools for decision support in the tactical and operational business processes of the industry, to provide a starting point for applying operations research and integrating decision support into their planning processes.

1.2 Contribution

The motivation and the contribution of this thesis is twofold: Firstly, to contribute to basic research on liner shipping network related optimization problems in liner shipping. Secondly, to identify tangible planning problems in the liner shipping industry, where operations research may provide good decision support to achieve better panning of the liner shipping transportation network.

1.2.1 Contributing to basic research on the LSNDP

The contribution of the thesis to basic research on the LSNDP is first and foremost, an article with a domain description of the liner shipping business. The domain description takes its offset in the business understanding of a leading global player, Maersk Line, and relates this to the field of operations research. The article presents an introduction to the problem, the related literature, a reference model and a heuristic column generation algorithm to provide the first computational results for the benchmark suite *LINER-LIB 2012*. *LINER-LIB 2012* is a public data set with realistic benchmark instances accompanying the article to ensure that data is available to researchers without an industry affiliation. The benchmark suite will hopefully enable researchers within the field to compare progress within mathematical modeling and algorithmic design. It has already been distributed to research groups in Germany, Holland, Norway and Singapore and we hope they will benefit from the data sets in their research. The full benchmark suite paper including a computational study constitutes Chapter 2.

The thesis explores two models for the LSNDP inspired by rich Vehicle Routing Problems as an alternative to multicommodity capacitated network design. A conference paper (Jepsen et al., 2011) with a new approach to modeling was published at the ICOR 2011 Conference. The model is a Dantzig-Wolfe decomposition on the set of services, where the pricing problem is to construct routes with cargo patterns. An extended version of the paper elaborating on the complexities of the pricing problem may be found in chapter 6. In Chapter 7 this model is extended to decompose per vessel class resulting in a different pricing problem, which may prove to be less complex than the pricing problem of Jepsen et al. (2011). Whether these models will be a competitive alternative to existing models on the LSNDP remains an open research question.

At present exact methods are not able to solve realistic sized instances of liner shipping network design and to this end heuristic methods can be pursued in order to solve real-life problems of the LSNDP in reasonable time. The benchmark article presents a metaheuristic for solving the benchmark suite instances. The computational results are promising, but it is still a challenge to construct a heuristic method, which scales in terms of the neighbourhoods searched and the evaluation of a given network. In this thesis we also present a matheuristic for the LSNDP targeting the performance of neighborhoods and network evaluation. A large scale neighborhood is defined as an integer program considering simultaneous insertion and removal of several port calls on a single service considering the multicommodity flow of the entire network. The integer programs for each service is embedded in a local search framework. The approach is similar to the ones seen in (Chen et al., 2007; Gulczynski et al., 2010, 2011). The method present the first results requiring a strict weekly frequency for all services and all vessel classes. This means that the

solution space is more restricted than the solution space of the algorithm in chapter 2 introducing both weekly and biweekly frequency and the objective values are thus generally lower than the ones presented in Chapter 2. The method finds profitable solutions for 5 out of 7 instances and has a good performance for most instances in terms of the execution time. Some of the ideas for increasing the performance of neighborhoods and evaluating a candidate solution may be refined and exploited in future metaheuristics for the LSNDP.

1.2.2 Decision support for network design in the liner shipping industry

In this thesis we propose three approaches that may prove as good business cases, where minor modifications to current practise may improve decision making of liner shipping network design. Firstly, there is the revenue management booking model considering empty repositioning in Chapter 3. This application enables a network planner to route cargo on the existing network choosing the most profitable cargo while simultaneously planning the redistribution of empty containers to points of demand. Secondly, the Vessel Schedule Recovery Problem in Chapter 4 considers different combinations of recovery techniques to mitigate the operational cost of disruptions. Lastly, the matheuristic for the LSNDP presented in Chapter 5 is designed such that a subset of services may be optimized upon considering the entire network. This tool may be used to reoptimize the network based on changes to demands, related services etc.

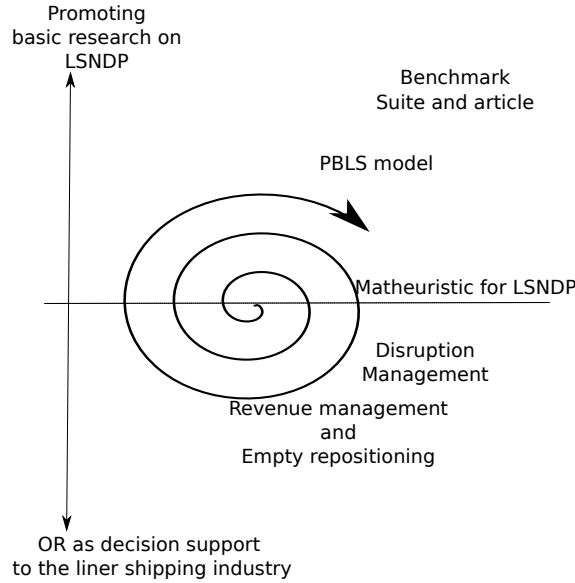


Figure 1.1: Placing personal motivation with thesis contributions. At the top we see basic research drivers such as modeling the problem, defining relevant data sets and describing the domain. At the bottom we see the industry drivers of creating tangible operational problems, that may benefit from decision support tools. The matheuristic is placed in the middle, because it may serve as a solver for the LSNDP as well as being used for fine tuning an existing network.

The motivation behind this thesis and the contributions presented are illustrated in Figure 1.1. The matheuristic for LSNDP is placed in the middle as it has potential to become a heuristic method for creating liner shipping network designs using a construction heuristic, but may also be used as a decision support tool, with an existing network as input, fixing optimization to a certain area of the network, to make smaller adjustments to the network given a new market or fleet situation.

1.2.3 PhD Thesis overview

The thesis is organized as follows:

Part I: Introduction. This part consists of two chapters. The current chapter contains a motivation and background for the thesis. Chapter 2 is the submitted version of an article accompanying the benchmark suite of data instances.

- **A base integer programming model and benchmark suite for liner shipping network design** This paper is an introduction to the LSNDP and the goal of the paper is to create an organized platform for the LSNDP by reviewing existing literature within the field and providing a thorough domain description seen from the perspective of a global liner shipping company, Maersk Line. A benchmark suite of data instances is presented. Data sources and methods to transform the data into the data instances are explained in detail. It additionally entails a reference model of the LSNDP which is an extension of the model from Alvarez (2009). A heuristic column generation algorithm is presented and the first computational results for the accompanying public benchmark suite are reported. The work has been presented as follows:
 - A paper co-authored with José Fernando Alvarez, Christian E.M. Plum, David Pisinger and Mikkel M. Sigurd is accepted with minor revision in *Transportation Science* (Brouer et al., 2012a).
 - Presentation at the EURO XXV, Vilnius, Lithuania, 2012 (presenter: Berit Dangaard Brouer).
 - A technical report of the initial work is published at DTU Management Engineering (Løfstedt et al., 2011).

Part II: Topics for optimization in liner shipping. The part consists of three papers, which may prove as good business cases for decision support in the industry.

- Chapter 3: **Liner Shipping Cargo Allocation with Repositioning of Empty Containers** present the first business case as a network revenue management tool to maximize the profit of a given network considering the cost of repositioning empty containers due to the global trade imbalance. The model is formulated as an extended multicommodity flow problem with interbalancing constraints. An arc flow model is decomposed to a path flow model using Dantzig-Wolfe decomposition and a delayed column generation algorithm is presented. A simple integer rounding heuristic is applied to obtain integer solutions. Computational results are reported for seven instances based on real-life shipping networks. Solving the relaxed linear path flow model with a column generation algorithm outperforms solving the relaxed linear arc flow model with the *CPLEX* barrier solver even for very small instances. The proposed algorithm is able to solve instances with 234 ports, 16278 demands over 9 time periods in 34 minutes. The integer solutions found by rounding down are computed in less than 5 seconds and the gap is within 0.01% from the upper bound of the linear relaxation. The work has been presented as follows:
 - A paper co-authored with David Pisinger and Simon Spoorendonk has been published in the *INFOR special issue on Maritime Transportation* Brouer et al. (2011).
- In Chapter 4 a disruption management tool is presented, which enables optimization of different recovery techniques related to a given disruption scenario. A time-space graph interpretation of the problem is described and a MIP model denoted the Vessel Schedule Recovery Problem (VSRP) is presented. The model evaluates a given disruption scenario and selects a set of recovery actions balancing the trade off between increased fuel consumption and the impact on cargo in the remaining network. The VSRP is proven to be \mathcal{NP} -hard and computational results for four real life cases and a number of generated scenarios is presented. In practise the method is efficient and solutions to the real life cases is found

using *CPLEX* within 5 seconds with comparable or improved solutions compared to realized recoveries of the real life cases. The work has been presented as follows:

- A paper co-authored with Jakob Dirksen, Christian E.M. Plum, David Pisinger and Bo Vaaben has been accepted for publication in European Journal of Operations Research (Brouer et al., 2012b).
- Presentation at the EURO XXV, Vilnius, Lithuania, 2012 (presenter: Berit Dangaard Brouer).
- In Chapter 5 we present a matheuristic, an integer programming based heuristic, for the LSNDP. The heuristic applies a novel greedy construction heuristic based on an interpretation of the liner shipping network design problem as a multiple quadratic knapsack problem. The construction heuristic is combined with an improvement heuristic with a neighborhood defined by the solution space of a mixed integer program. The mixed integer program optimizes the removal and insertion of several port calls on a liner shipping service. The objective function is based on evaluation functions for revenue and transshipment of cargo along with in/decrease of vessel- and operational-cost for the current solution. The evaluation functions may be used by heuristics in general to evaluate changes to a network design without solving a large scale multicommodity flow problem. The matheuristic targets some of the performance issues related to designing an efficient heuristic for the liner shipping network design problem. The work has been presented as follows:
 - A conference paper co-authored with Guy Desaulniers appeared in the Proceedings of The International Conference on Logistics and Maritime Systems 2012 (Brouer and Desaulniers, 2012) and was presented at the conference (LOGMS 2012) (presenter: Berit D. Brouer).
 - Presentation at the European Summer Institute on Maritime Logistics (ESI2012), Bremen, Germany, 2012 (presenter: Berit Dangaard Brouer).

Part III: Modeling the Liner shipping Network Design Problem consists of two chapters, where the first is an extended version of a conference paper.

- Chapter 6 is a revised version of **A Path Based Model for a Green Liner Shipping Network Design Problem** elaborating on the complexity of the pricing problem. The paper presents a new path based MIP model for the Liner shipping Network Design Problem. The proposed model reduces the problem size using a novel aggregation of demands. A decomposition method enabling delayed column generation is presented. The subproblems have similar structure to resource-constrained-shortest path problems, but due to the nature of a routing as a non-simple cycle, it becomes an open research question to solve the pricing problem to optimality. The work has been presented as follows
 - Presented at Optimization Days, GERAD & CIRRELT Montréal, (2011) (Presenter: Berit D. Brouer)
 - Published in Proceedings of The International MultiConference of Engineers and Computer Scientists (2011) and presented at the conference (2011)(presenter: Mads K. Jepsen)
- Chapter 7 is a general discussion and analysis of modeling issues in relation to the LSNDP with regards to current state-of-the-art and the related optimization problem of Multicommodity Capacitated Network Design and Vehicle Routing Problems. The model from Chapter 6 is reformulated to a decomposition per vessel class resulting in a different pricing problem, which could be easier to solve. However, there are open research questions with regards to the construction of models suitable for cutting planes and pricing in a Branch-and-Bound context.

Chapter 8 contains a summary of the thesis with concluding remarks and perspectives for future work.

Appendix B is an extended abstract on an early version of the construction heuristic used for the matheuristic in Chapter 5 presented at TRISTAN VII, Tromsø, Norway (2010)(Løfstedt, 2010). The project was presented at the conference. (presenter: Berit D. Brouer)

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Chapter 2

A base integer programming model and benchmark suite for liner shipping network design

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Abstract The liner shipping network design problem is to create a set of non-simple cyclic sailing routes for a designated fleet of container vessels, which jointly transports multiple commodities. The objective is to maximize the revenue of cargo transport, while minimizing the costs of operation. The potential for making cost effective and energy efficient liner shipping networks using Operations Research (OR) is huge and neglected. The implementation of logistic planning tools based upon OR has enhanced performance of airlines, railways and general transportation companies, but within the field of liner shipping applications of OR are scarce. We believe that access to domain knowledge and data is a barrier for researchers to approach the important liner shipping network design problem. The purpose of the benchmark suite and the paper at hand is to provide easy access to the domain and the data sources of liner shipping for OR researchers in general. We describe and analyze the liner shipping domain applied to network design and present a rich integer programming model based on services, which constitute the fixed schedule of a liner shipping company. We prove the liner shipping network design problem to be strongly NP-hard. A benchmark suite of data instances to reflect the business structure of a global liner shipping network is presented. The design of the benchmark suite is discussed in relation to industry standards, business rules and mathematical programming. The data is based on real life data from the largest global liner

¹Submitted and under revision at *Transportation Science* (2012)

shipping company, Maersk Line, and supplemented by data from several industry and public stakeholders. Computational results yielding the first best known solutions for 18 of the 21 benchmark instances is provided using a heuristic combining tabu search and heuristic column generation.

Keywords: liner shipping, mathematical programming, network design

2.1 A benchmark suite for research on liner shipping network design problems

Operations research (OR) is widely used within the transportation sector to provide a cost efficient and competitive organization. However, application of OR within containerized liner shipping is scarce (Christiansen et al. (2004)). The potential impact of OR on this billion dollar industry is enormous especially given the large concentration of players in the business. Maritime shipping produces an estimated 2.7% of the world's CO_2 emissions, of which 25% is attributable to container ships alone (The World Shipping Council (2009)). An energy efficient liner shipping network is becoming increasingly important to all stakeholders. OR can help in designing effective and energy efficient liner shipping networks to mitigate the carbon footprint of the liner shipping industry. We believe that the lack of OR within liner shipping is partly due to barriers for new researchers to engage in the liner shipping research community. Constructing mathematical models and creating data for computational results requires profound knowledge of the domain and data sources. The benchmark suite presented in this paper aims to make liner shipping network design problems (hereafter LSNDP) approachable for the research community in general. We wish to create a platform where methods can be compared on a set of known data instances. By disseminating our knowledge of the liner shipping domain into real world network data instances for mathematical programming, we hope to diminish this formidable hurdle. Our hope is to enable research on the field of liner shipping network design to develop like the Vehicle Routing Problems (VRP) have through the benchmark instances of Solomon (1987). The benchmark suite is seen as the root of a tree where new branches will appear as our ability to solve more complex interpretations of the liner shipping problem grows.

The initial model of the diverse field of VRP models was the Capacitated Vehicle Routing Problem (CVRP). We present a reference model for the LSNDP, which is an extended version of the model in Alvarez (2009). The reference model may be used for comparing various methods applied to the problem and serve as an inspiration to future model development. Our goal is to enable the benchmark suite to support future model and method development, and hence the data of the benchmark suite encompasses several attributes such as transit times not applicable to the reference model. In Section 2.2 relevant literature within the field of liner shipping economics will underpin the importance of various costs and restrictions within network design. A review of the literature within OR and the various models presented is provided. We review methods for solving liner shipping network design related problems and the computational results in the literature to illustrate the computational hardness of the problem. Section 2.4 provides an introduction to the liner shipping network design domain. The domain discussion is complemented by the strategic business and domain knowledge of one of the major global operators within liner shipping. We characterize the benchmark suite and delimit the data supplied in the benchmark suite to current mathematical models along with the open research questions we try to support with the data provided. In Section 2.5, we discuss the data objects and data generation. The goal for the instances generated is to capture real life cost structures, trade imbalances, market shares and scale for a global liner shipping company. We discuss the diversity of instances needed to qualify and challenge development of both exact and heuristic methods for solving the LSNDP. Finally, we solve the generated instances by using a new tabu search heuristic based on the framework by Alvarez (2009). The algorithm has been extended to handle more complex routes (so-called butterfly routes) and to ensure a (bi)-weekly time schedule. Moreover, an improved MIP neighborhood is used to generate new candidate routes. Computational results are presented in Section 2.6 to qualify the benchmark suite with experimental results and establish the currently

best known solutions for the liner shipping network design problem and hopefully spur competition and interest into our field of research. Section 2.8 contains conclusions and directions for further research into liner shipping network design problems. Sections 2.4 and 2.5 is a shortened version of the technical report Løfstedt et al. (2011). The reader is referred to this report for additional information.

2.2 Literature review

The two papers, Christiansen et al. (2004) and Christiansen et al. (2007) provide excellent surveys of OR in maritime transportation. They point to the limited research of planning problems in liner shipping in contrast with the size and the possible impact of optimization within the industry: There are a number of contributions on the individual economic and structural factors in designing liner shipping networks, but no comprehensive description of the general liner shipping domain related to network design has been published to date. One contribution of the present paper is to give an overview of these factors related to the LSNDP. In the first part, we give an overview of contributions related to some of the factors affecting network design such as *network configuration*, *bunker price*, *transit time*, *competitive position*, *repositioning of empty containers*, *frequency*, *port call sequence*, *schedule*, and *fleet deployment*. In the second part we review contributions on the LSNDP.

The *network configuration* of carriers is explored in Notteboom (2004) concluding that a global network will not have a pure hub-and-spoke structure or a pure multi-port structure. The economy of deploying super panamax vessels on either a multi-port or hub-and-spoke network structure is investigated in a case study by Imai et al. (2009). They conclude that the multi-port structure is superior for the Asia-North America and Asia-Europe trades, whereas the hub-and-spoke structure is advantageous in the European trades.

The impact of *bunker price* on the network configuration of liner shipping companies has been explored by Cariou (2010); Notteboom and Vernimmen (2009); Stopford (2009). Seen from the carriers' point of view, reducing speed and making effective use of the capacity deployed in the network has economical incentive as the general analysis of vessel costs by Stopford (2009) reveals bunker fuel as the dominant cost in operating a liner shipping network. According to Notteboom and Vernimmen (2009) managing the bunker consumption in the network gives the carriers strong incentive to reduce speed, deploy additional vessels to services, and increase buffer time in the schedule to avoid having to increase speed to accommodate for delays and port congestion. The incentive to reduce speed depends on the actual bunker price because the additional vessels deployed to maintain the frequency come at an increased capital cost. Cariou (2010) argues that a bunker price exceeding 350 USD per ton will ensure the sustainability of "slow steaming", which means sailing at a reduced average speed. Cariou (2010) calculates the CO_2 reductions from slow steaming from 2008-2010 to 11% attesting that slow steaming is a very effective way of reducing the carbon footprint of the liner shipping industry. Recently, Wang and Meng (2012a) investigate sailing speed optimization for each individual port-to-port voyage on a liner shipping service to reduce bunker consumption using an outer-approximation method.

In liner shipping there is an inherent trade off between reducing bunker consumption through speed reduction and achieving competitive *transit time* for cargo. Notteboom (2006) explores the time factor in liner shipping network design related to transit time of a cargo routing and schedule reliability. An important analysis on the relation between the number of port calls on a service and competitive transit times is reported. The conclusion is that the time spent at ports is very significant and hence the number of port calls is decisive for the transit time of direct connections at the end points of a service. Wang and Meng (2011) evaluate the schedule design along with container routing for a fixed network with predefined paths for container shipments in order to minimize transshipment cost and transit time.

Gelareh et al. (2010) explore the possible *competitive positions* of a carrier in a market with an incumbent carrier. The relation between time and cost on the market share is modeled and investigated. Gelareh et al. (2010) underpin transit times and level of service as important factors

in the construction of a liner shipping network design. Wang and Meng (2012b) propose a tactical model for schedule design capturing the trade off between speed optimization and transit time levels taking into account time uncertainty at sea and in the port times. We believe there will be a surge in sailing speed optimization taking level of service and schedule reliability into account where the liner shipping service network has already been fixed to a large degree. We are not aware of published models of the LSN DP incorporating distinct cargo transit times or level of service, but we believe that future research on the LSN DP will incorporate these factors.

Another important factor in a liner shipping network is *repositioning of empty containers* due to trade imbalances. Shintani et al. (2007) design a model for a single service of a carrier. The experimental results indicate that empty repositioning is significant for the port calling sequence and the cargo handling costs incurred. Dong and Song (2009) investigate the proportion of empty repositioning given current global container trade using the existing global liner shipping network. Their results conclude that 27% of all container traffic is empty repositioning. Brouer et al. (2011) show that joint optimization of demanded cargo and empty repositioning in a fixed network is viable. Meng and Wang (2011) incorporate empty repositioning and port productivity into the evaluation and selection of candidate shipping routes. Empty repositioning is so far not incorporated into the cargo allocation in models, where the set of candidate routes are not fixed, as in the LSN DP.

Liner shipping and the industry generally adhere to a schedule based on a *weekly frequency of port calls* grouping vessels of similar characteristics onto services (Notteboom, 2004, 2006; Stopford, 2009). In Alvarez (2012) the cost of the practice of a weekly frequency is questioned, and level of service expressions are formulated in order to investigate the impact of maintaining a weekly frequency as opposed to another fixed frequency. Several contributions on the optimal *port calling sequence* may be found for a single route, e.g., Chu et al. (2003); Hsu and Hsieh (2007); Lu (2002); Shintani et al. (2007)

Scheduling of liner shipping services is a related optimization problem. An example of scheduling with fixed port call sequence may be found in Yan et al. (2009) as well as the more recent papers of (Wang and Meng, 2011, 2012b), whereas Agarwal and Ergun (2008a) decides scheduling to a certain weekday and routing simultaneously.

The fleet deployment problem (FDP) is closely related to LSN DP as fleet deployment is often implicitly considered in the network cost. FDP assumes that the service in terms of actual vessel voyages is fixed and hence that the routing is already decided upon. FDP decides on how to assign the carrier's own vessels to the actual vessel voyages along with the options of chartering in (leasing) vessels or chartering out (forward leasing) own vessels to minimize the cost of maintaining the schedule. Fleet deployment is generally described in Christiansen et al. (2007) with reference to the main paper within fleet deployment (Powell and Perakis, 1997). Fagerholt et al. (2009) is a recent paper and case study on liner shipping fleet deployment with Höegh autoliners as a case study. The recent paper by Gelareh and Meng (2010) considers a liner shipping fleet deployment problem, where the frequency of sailing is decided upon in relation to demand coverage and speed on the individual voyage. The paper gives a thorough review of fleet deployment literature to date. The model is solved using a Mixed Integer Programming (MIP) solver for three and four services using three to five vessel types. Recently, the fleet deployment problem in conjunction with transit time levels are described by Wang and Meng (2012c) expanded with weekly frequency considerations in Meng and Wang (2012).

Literature on models and methods for *liner shipping network design* within OR is still limited although recent years show increasing activity in the field. Kjeldsen (2011) provides a classification scheme for routing and scheduling within liner shipping. The classification scheme entails 24 references in total for routing and scheduling problems within liner shipping. The classification scheme of Kjeldsen (2011) show that there is a large variance in the models with respect to cost structures, constraints and scope unveiling that we are dealing with a young research field with many possibilities of scoping network design to various decisions such as scheduling, fleet sizing, fleet deployment, route generation, speed optimization and many more.

An early contribution is that of Rana and Vickson (1991), who present a model for liner shipping with non-simple routes by using a head- and back-haul structure. The model does not allow

transshipments, which is at the core of liner shipping today. Fagerholt (2004) develops a model and solution method for a regional carrier along the Norwegian coast. The model assumes the carrier loads at a single port and finds optimal routes of vessels to service the unloading facilities. The problem may be dealt with as a VRP problem, given that a designated depot is known and transshipments are not allowed. The solution method is a MIP solver. Similarly, Karlaftis et al. (2009) solve a problem for the region of the Aegean sea using a genetic algorithm related to the coastal freight problem described in Sambracos et al. (2004). These models do not deal with the important concept of transshipments at multiple ports and the resulting interaction between different services.

Agarwal and Ergun (2008a) are the first to include a *weekly frequency constraint* by grouping vessels into vessel classes in the *simultaneous ship scheduling and cargo routing model*. The model creates routings for a set of vessel classes with a rough schedule. The weekly frequency constraint decides the number of vessels deployed to a service according to the service duration, and they introduce a time-space graph spanning each weekday to reflect the availability of transportation on a certain weekday. The model allows for transshipments, but the cost of transshipments cannot be derived and hence the overall cost structure excludes the important cost of transshipment. The time-space graph gives a temporal aspect in terms of a rough schedule based on weekdays, but the time-space graph is not utilized to reflect transit time restrictions. Blander Reinhardt and Pisinger (2012) present a model, which is one of the first to include transshipment cost and present a branch-and-cut method for solving a liner shipping network design problem with butterfly routes to optimality. The model selects a route for each individual vessel in a fleet. This results in a distinct network layer for each vessel and the grouping of vessels to a single service is not explicit in the model. The configuration is suitable for smaller carriers. Global carriers tend to group a set of vessels with similar characteristics to a single service to reduce the complexity of the network design (Notteboom and Vernimmen, 2009; Stopford, 2009) and to have customer-friendly schedules.

Alvarez (2009) is the most recent publication on LSNDP considering *the joint routing and fleet deployment model*. It bundles a service with a vessel class, the number of vessels deployed to the service, a target speed, and a non-simple cyclic port sequence. The cost of a service in the model reflects the deployment cost of the vessels in the service and the estimated bunker consumption adjusting for the difference in bunker consumption, when sailing at different speeds or idling at a port. The total fleet cost accounts for whether vessels are deployed to services or are chartered out at market prices. Cargo revenues and handling costs are accounted for in the model along with a penalty for cargo that has been forfeited due to insufficient capacity or insufficient revenue in the view of total network cost. The model assumes a planning horizon with stable demands and does not impose restrictions on the frequency of service or the actual scheduling of the services. A case study with discussion on data generation and port/vessel class incompatibilities is provided. The transshipment cost of non-simple routes is, however, not calculated coherently, as the model cannot detect such a transshipment.

Agarwal and Ergun (2008a) prove the NP-completeness of *the simultaneous scheduling and cargo routing problem*. The LSNDP in general may be viewed as a VRP with split-pickups, split-deliveries, multiple cross docking, no depots and a heterogeneous vessel fleet and hence the LSNDP is strongly NP-hard as proven in section 2.6.4. The methods deployed to solve the problem are mainly heuristics based on integer programming and decomposition techniques. The branch-and-cut method of Blander Reinhardt and Pisinger (2012) and a MIP formulation of Alvarez (2009) solve smaller instances to optimality. Remaining solution methods are heuristic methods. An overview of the reviewed methods on liner shipping network design is given in Table 2.1.

Computational results are scarce and based on individual data sets, but it is clear from the literature that solving large scale instances is a hard task even for heuristics. The benchmark suite presented in this paper will aid comparison between different methods if the public benchmark data is used for computational results. In Christiansen and Fagerholt (2011) the importance of creating public benchmark instances for maritime transportation problems is advocated in order to drive basic research on maritime optimization problems forward.

Alvarez (2009) identifies several problems with exact optimization methods. Firstly, the feasible

Article	Routes	Fleet	Routing	Method	Optimal	Constraints	vessels/ports
Rana and Vickson (1991)	multiple	heterogen	in/out bound	Lagrange, Benders	No	Capacity, time, connected	3v, 20p
Shintani et al. (2007)	single	vessel class	in/out bound	Genetic algorithm	No	Empty repositioning, bunker cost and handling cost	3v, 20p
Karlaftis et al. (2009)	multiple	heterogen	simple cycle - VRP	Genetic algorithm	No	capacity and time windows on demands	not fixed, 26 p
Fagerholt (2004)	multiple	heterogen	simple cycle - VRP	Route generation/ MIP solver	Yes	capacity, route duration	20v, 40p
Blander Reinhardt and Pisinger (2012)	multiple	heterogen	butterfly cycle	Branch-and-Cut	Yes	capacity, time, connected, transshipment	6v, 15p
Agarwal and Ergun (2008a)	multiple	vessel class	simple cycle	Greedy, column generation, Benders	No	weekly frequency, transshipment	50v, 10p
Alvarez (2009)	multiple	vessel class	non-simple	MIP solver/tabu search	Yes/No	transshipment, bunker	100v, 7p (MIP solver), 120p (Tabu search)

Table 2.1: Overview of the models on liner shipping network design published in international journals. *Routes*: refers to whether the problem is defined for a single or multiple routes/services. A *Fleet* is homogeneous if only a single type of vessel (in terms of capacity and technical specifications) and heterogen for multiple vessel types. *Routing* denotes the way routes are perceived and hence constructed by the model, see section 2.4.2. *Method* refers to the algorithmic solution method, *optimal* to whether or not the method produces an optimal solution, *Constraints* are a summary of the constraints included. Lastly, *vessels/ports* states the number of vessels and ports in the computational study.

solution space of a single rotation is vast. Combining this with a vector of different capacities and cost for every feasible rotation increases the feasible solution space significantly. Due to economies of scale, a solution to the LP-relaxation will prefer to use a fraction of large vessels, leading to highly fractional solutions. This is due to a cost structure divided between the vessels and the cargo flow for models incorporating transshipment, but the revenue follows the cargo flow. Therefore, convergence of decomposition techniques such as column generation might be slow.

Recent developments in modelling the LSN DP have tried to overcome this division by coupling service generation with the cargo load as seen in Jepsen et al. (2011) and Kjeldsen (2012). No computational results are reported for Jepsen et al. (2011) as the pricing problem has a very complex structure, which cannot be solved efficiently using current state of the art methods such as resource constrained shortest path problem. Kjeldsen (2012) solve the pricing problem heuristically and some limited computational results are reported, which confirm the hardness of this problem even for heuristic methods.

2.3 Contribution

The present paper gives an overview of the domain of liner shipping network design related to mathematical formulations of the LSN DP and other network related optimization problems. It is evident that modelling the LSN DP in a compact formulation, which can exploit state-of-the-art mathematical programming techniques is still a challenge for our community and we expect further development in this area. In this paper we present a reference model, which is an extension of Alvarez (2009), where transshipment on butterfly routes is correctly accounted in the objective function without introducing increased complexity and (bi)-weekly frequency is imposed in the route generation. The model presented is a simplified, but realistic description of the LSN DP as described in this paper. We believe that the model along with the benchmark suite can create a platform for the development of heuristic and exact solution methods. We provide a heuristic solution method for solving the LSN DP and the first computational results for our benchmark suite. Any model adhering to the constraints and the objective of the reference model of this

paper, should be able to compare itself to the computational results presented and improve the research direction of the LSNDP. The benchmark suite can cater for extensions such as service level requirements, port productivity and sailing speed optimization to cater for future research directions within the field of mathematical models of the LSNDP. The benchmark suite is also applicable to other related and promising research problems within liner shipping network design, where the routes or the cargo flows have been fixed to some extent as seen in, e.g., (Meng and Wang, 2011; Wang and Meng, 2011).

2.4 Domain of the liner shipping network design problem

The market and business of liner shipping is thoroughly described and analyzed in Stopford (2009) and Alderton (2005). Complemented by the experience of network planners and optimization managers at Maersk Line, this section introduces the business context of liner shipping network design.

2.4.1 The liner shipping business

The business of liner shipping is often compared to public transit systems such as bus lines, subways and metro. A *service* is a *round trip* sailed at a given *frequency*. The schedule can be consulted to find the next call of the service at a port. The *round trip* is the set of designated stops, which for a liner shipping company is a set of *ports*. The round trip in liner shipping may be divided into a *head* and a *back* haul to distinguish between different demands according to their regional specifications. The head haul is usually the demand intensive direction. Some services are central to connecting many origins and destinations, whereas others serve a smaller distinct market. In liner shipping a distinction is made between *trunk services* serving central main ports with several demands and *feeder services* serving a distinct market typically visiting a single main port and a set of smaller ports. Like in public transit, a transport may include the use of several services to connect between the origin and the destination of the transport. In liner shipping we refer to transits as *transshipments*. A *fleet* of *vessels* with varying capacity and speed is deployed to the services according to the demand. The *transit time* of a cargo denotes the time a cargo travels from origin to destination. Transit time is counted in days and transit times may vary from a single day to 90 days.

2.4.2 Network

The network must be *competitive* and *efficient*. A *competitive* network may accommodate several routings for one origin-destination pair varying on transit time and cost. Most global liners provide several itineraries for an origin-destination pair by end-to-end services as well as transshipment services with different transit times and freight rates. In order for a network to be *competitive* it must offer low transit times and few transshipments. A *competitive* network serves the main ports of a region frequently with good connections to feeder ports with a high schedule reliability. An *efficient* network facilitates transshipments at terminals with high crane productivity and container capacity to minimize the cargo handling time. Transshipments are also used to get effective utilization of vessel capacity such that the trunk services are fed by several feeder services and direct cargo along the trunk line. Finally, empty containers must be effectively repositioned to ensure availability of containers at the origins of the cargo. This is especially critical for reefer containers as several regions such as South America and New Zealand have a large export of refrigerated goods, but an insignificant import of refrigerated goods. Examples of different liner shipping networks are seen in Notteboom (2004), which are included in Figure 2.1. Notteboom (2004) argues that global liners are multi-layered networks of different types since they are all competitive for particular circumstances.

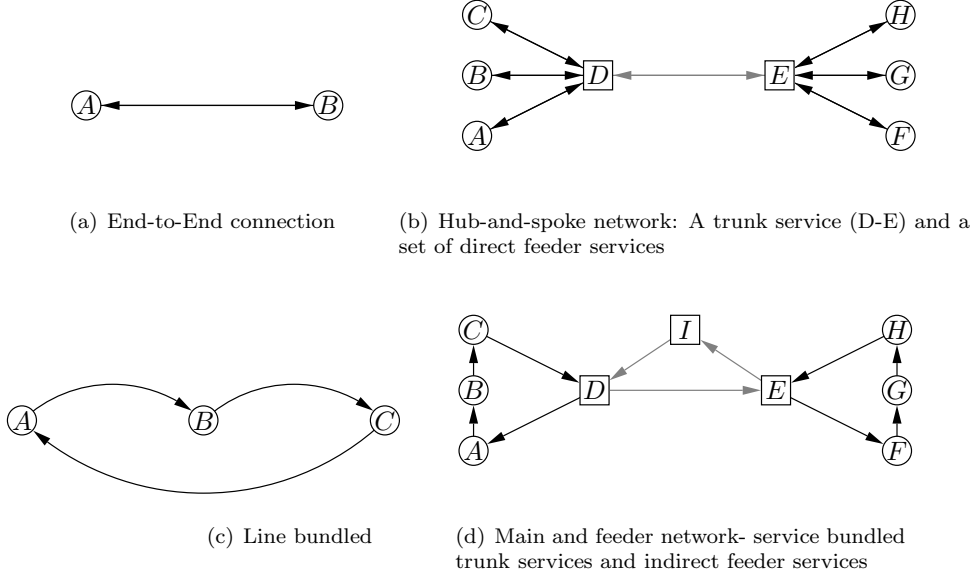


Figure 2.1: Examples of routes and network designs. Square nodes symbolize Hubs or main ports, whereas round nodes symbolize feeder ports.

Value propositions

A specific liner shipping company is referred to as a *carrier*, whereas the owner of a certain cargo is denoted a *shipper*. The competitive position of a carrier is a combination of port coverage, price, transit time, transshipments, schedule reliability, and, in recent years corporate social responsibility of the company, where environmentally friendly transport has received a lot of attention. The *freight rate* of transporting some cargo depends not only on the actual network cost, but also on the transit time, the container type needed (e.g., a refrigerated container), special regulations (restricted and dangerous goods), the number of transshipments, flag of the vessel(s), and naturally the relation between demand and supply for cargo transport on the connection in question. In addition the price covers the administrative overhead incurred by the carrier comprising approximately 30% of total cost (Stopford, 2009). Notteboom (2006); Notteboom and Vernimmen (2009) and Stopford (2009) discuss the importance of transit time in pricing cargo transportation as the inventory holding cost is paid by the shipper until the cargo is delivered (Notteboom, 2006). Furthermore, some goods are perishable and the time to market becomes crucial. It is becoming increasingly important for companies to ensure transport with the lowest CO_2 emission and hence slow steaming (in particular on back haul connections) to reduce network bunker consumption and overall CO_2 emission.

Forecast and planning horizon

A demand forecast for a given planning horizon is crucial to network design as the ideal network has a perfect fit between demand and capacity (Stopford, 2009). The demand for container transport fluctuates over a year with seasonal variance and peaks at certain times of year. These peaks may be *regional*, if they are related to a crop (e.g., bananas or lemons) or *global* (e.g., Christmas). Hence, the network design is rarely stable over a yearly period. Some structure is fairly stable, but additional structure will reflect seasonality and hence the planning horizon for a network is important to the liner service network design and to the fleet deployment. General economic and financial conditions have a major impact on the liner service network design and fleet deployment, but are hard to predict compared to a seasonal pattern, which may be recognized and accounted

for.

Service

A service from the carriers' point of view consists of a set of port calls with designated berthing times at a given frequency. The individual services are required to be good components of the network with regards to efficient transshipment facilities and fast direct services for critical connections. Notteboom (2004, 2006) categorize a wide range of rotation patterns as follows:

End-to-End: A direct shuttle service between 2 ports (see Figure 2.1). *Line Bundled*: A rotation visiting a set of ports in a loop. *Trunk Services*: An end-to-end connection between hubs. *Direct feeder services*. Shuttle from feeder port directly to a hub. *Indirect feeder services* - a service bundled rotation to a set of feeder ports and a hub. *Round-The-World (RTW)*: A rotation following the equatorial belt visiting hubs in order to service the east-west and north-south trades in a grid. This type of service has a capacity constraint as it must traverse the Panama canal restricted to panamax ships. *Pendulum*: A service traveling back and forth like a pendulum, e.g., Europe - Far East - US west coast - Far East - Europe. *Butterfly*: Multiple cycles centered around one port (Figure 2.2). Each cycle visiting an alternating sequence of ports. One cycle may be a subset of another. *Conveyor Belt*: A service connecting regional hubs designed for transshipments between continents at the crossing point of trade lanes.

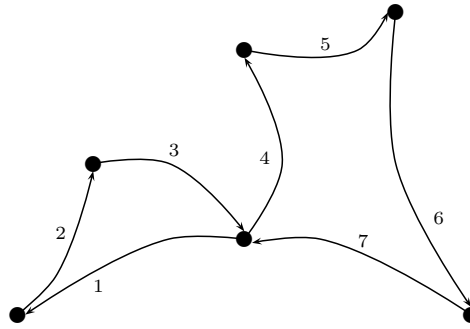


Figure 2.2: An example of a butterfly rotation

Rotation turnaround time varies from a single week up to 20 weeks, although the average rotation is around 8-9 weeks. The rotation turnaround time is composed of voyage time at sea and the service time at ports used for piloting in and out of the port, berthing and loading/unloading cargo. The number of port calls in a rotation is a trade-off between economies of scale and transit times (Notteboom, 2006). Visiting many ports leads to a good utilization of the vessel capacity at the cost of long transit times. Another complicating factor is that port stays are very time consuming. Notteboom (2006) reports that 21% of the transit time is the accumulated port stay for a COSCO Europe-Far East service. The schedule includes buffer time to account for delays due to weather, port congestion or unusually high terminal handling time. These delays may cause speed increases on individual voyage legs, which increases energy consumption and CO_2 emission.

Frequency

As described in Section 2.4.1, the liner shipping business is characterized by having a public schedule. From the carrier's point of view a schedule consists of a set of *services*. The services consist of a fixed itinerary of ports typically called with a weekly frequency (Notteboom and Vernimmen, 2009; Stopford, 2009). The fixed weekly frequency introduce constraints on the network as opposed to varying frequencies of services. The fixed weekly frequency is widely used in container liner shipping due to significant planning advantages for carriers, shippers and terminals:

- **Reliability.** Fixed departures enabling complete integration of customer supply chains.

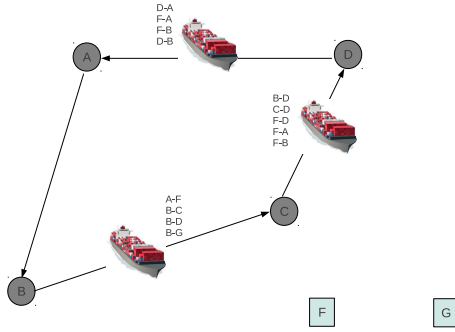


Figure 2.3: A single service connecting ports A,B,C and D. The vessels are depicted along with a cargo list specifying the current cargo load. Total round trip time is 21 days and to provide weekly frequency three vessels are sailing a week apart. The cargo is destined for nodes on the service as well as nodes not belonging to the service. The cargo has multiple origins and destinations.

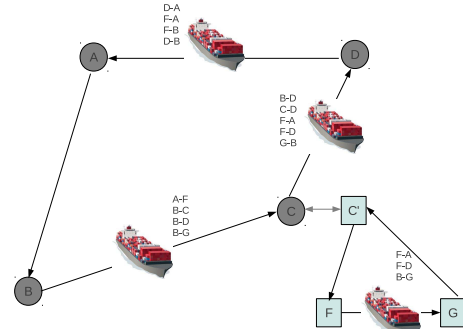


Figure 2.4: Two connecting services. The service from Figure 2.3 and a second service with a round trip time of 1 week illustrated by a single vessels. The cargo composition on board vessels illustrate transshipments at the core of the liner shipping network design.

- **Simplified network planning.** A guiding rule for the carrier when designing the network.
- **Asset planning.** The use of port berths and vessels can be planned for better utilization.
- **Planning routing scenarios.** Synchronization of connecting services may provide timely transshipments for critical connections.

The stable flow of general cargo in containers enables the carrier to maintain a weekly service. The weekly frequency of a schedule is achieved by deploying multiple vessels sailing one week apart as illustrated in Figure 2.3. The speed of each voyage between port pairs is thus closely related to the number of vessels. Therefore, adjusting the number of vessels on a service is a means to adjust speed. At the same time, the decision of speed enables the carrier to adjust to demand fluctuation as the carrier can increase capacity by increasing speed.

Business rules for service generation

The generation of services has a high degree of freedom in terms of the number of port calls and average rotation time. In spite of the route flexibility there are certain business restrictions arising from complex real world constraints. The real world constraints may be expressed as “*rules of thumb*” to the combination of the vessel class deployed, the rotation turnaround time and degrees of freedom on the weekly frequency, according to Network and Product at Maersk Line at Maersk Line. Some important business rules are stated as follows:

1. Services deploying a vessel class of capacity of at least 1200 *Forty Feet Equivalent Unit*(FFE) must have weekly frequency. This is a basic structure in *liner* shipping, customers plan their production after a weekly scheduling. It also facilitates easy management of shared resources between carriers as terminals and canals. Smaller vessels calling smaller ports are exempt from this due to low demand and / or unstable terminal service.
2. Services deploying a vessel class of capacity of at least 4200 FFE must have at least 4 week rotation time. Large vessels have longer port operations, thus using these on short services would give a high fraction of port time on the rotation. This again gives a high transit time as compared with smaller vessels on same rotation, thus making large vessels on short services uncompetitive.

3. Services deploying a vessel class of capacity of at most 800 FFE must have at least 2 feeder portcalls in between every hub-port call.

Business rules 1 and 2 are implemented in our algorithm and thus present in the computational results. Business rule 3 has not been implemented as distinguishing the sequential mix of hub and feeder port calls is outside the scope of the heuristic column generation algorithm.

2.4.3 Assets and Infrastructure

Cost structure and economies of scale

The network cost of a carrier can be divided into fleet cost and cargo handling cost, detailed below (we omit the administrative overhead estimated to around 30% by Stopford (2009)):

- **Fleet Cost:**

1. *Bunker Cost* is the cost of bunker, which is the fuel consumed by container vessels.
2. *Capital cost* is the cost of acquiring or financing a single vessel v .
3. *Port Call Cost* is a terminal fee for calling a terminal with a given vessel v . (See below for costs related to cargo handling at ports).
4. *Canal Cost* is the cost of traversing a canal with a given vessel.
5. *Operational Cost (OPEX)* is the operating cost of a vessel including crew, maintenance and insurance.

- **Cargo handling Cost:**

1. *(Un)load cost* is the cargo handling cost at a given port.
2. *Transshipment cost* is the cost of transshipping a cargo in a port.
3. *Equipment cost* is the cost of owning / leasing containers.

According to Stopford (2009, Table 13.9), the bunker cost is 35-50% of a vessel's cost, capital cost is 30-45%, OPEX is 6-17% and port cost 9-14%. Generally bunker cost exceeds capital costs (apart from the largest vessels).

Bunker consumption depends on the vessel type, the speed of operation, the draft of the vessel (e.g., the actual load), the number of operational reefer containers powered by the vessel's engine and the weather. Bunker consumption for a vessel profile is a cubic function of speed (Alderton, 2005; Stopford, 2009) During a round trip the vessel may sail at different speeds between ports. The vessel may *slow steam* to save bunker fuel or increase speed to meet a crucial transit time. Speed may be constrained by hard weather conditions or navigation through difficult areas. This is a complicating aspect seen from a modeling perspective.

Both capital cost and OPEX varies with capacity. Economies of scale means that a large vessel is cheaper to operate per Forty Foot Equivalent (FFE), which is the most common container unit (Stopford, 2009). The market rate of a vessel is called *Time Charter Rate (TC rate)* and represents the cost of leasing (charter in) a container vessel into the fleet or for a carrier to forward lease (charter out) an owned vessel to another carrier. The TC rate fluctuates with seasonality and is highly dependent on the length of the chartering period. A carrier may have an owned fleet supplemented by chartering in and out to meet capacity requirements and to gain flexibility in asset management. TC cost will include daily running costs (OPEX) of the vessel, such as crew costs, repair and maintenance. For vessels owned by the carrier the TC cost will cover OPEX and capital cost and depreciation of the vessel's value (see Stopford, 2009, page 544). It is assumed that the TC costs represent a market rate, where the carrier will be able to charter out the vessel in case of a surplus of vessels. The methods described herein will consider fixed TC costs, not considering financial asset management of a fleet of container vessels under an expected development of TC costs. For more details on such ideas see (Alvarez et al.).

Port call cost and canal costs may be treated identically. The port call cost is a fee paid to the terminal. The fee depends on the size of the vessel, i.e., the capacity booked in the terminal and also on the geographical location and size of the terminal. The cost of traversing the Suez Canal and the Panama canal depends on the size of the vessel and the actual load of the vessel.

(Un)load and transshipment cost are also known as *cargo handling costs*. The load and unload costs are fixed once the cargo is selected for transport, but the transshipment cost depends on the routing of the product and hence the total number of transshipments. A global carrier will not provide direct connections for a significant percentage of the available transport scenarios, which incur transshipment at least once during a voyage. The cargo handling cost can be a non-linear function as some terminals have volume dependent costs.

Equipment cost is the cost of containers. The carrier usually owns the majority of the containers used for cargo transport with the option of leasing in containers (Stopford, 2009). Stopford (2009) estimates the daily cost of a FFE dry container to around 1\$ per day, whereas a reefer FFE has a daily cost of around 5.60\$.

Vessels

An ocean-going container vessel is the core part of a carriers' operations. It can be characterized by specifications as *FFE capacity, weight capacity, speed, length, beam, draft, number of reefer plugs, ice class, age, engine power, etc.* The defining attribute is FFE capacity given as a nominal number. The actual capacity of a vessel depends on the service it is sailing and the actual cargo on board. A vessel cannot accommodate more reefer containers than it has reefer plugs. The draft, length and beam of the vessel dictate which ports, canals and straits the vessel can access. Some waters have special access restrictions like the gulf of Finland, which during winter requires ice class vessels for service. With regards to network design, the vessels are grouped in vessel classes with similar properties such as capacity and speed interval, e.g., a Panamax vessel class denoting the maximal width for traversal of the Panama Canal. Other common groupings are according to a capacity band, i.e., vessels with a nominal capacity of 1500-2100 TEU.

Each vessel has a minimum speed S_{min} , and a maximum speed S_{max} , in knots, and a *design speed* S^* , and *design draft* at which its design fuel consumption F^* , is optimized. Still the actual speed largely decides the fuel consumption. Large vessels may use in excess of 200 metric tonnes (mt) of fuel per sailing day. Additional fuel is consumed by auxiliary engines for other vessel systems (1-12 mt/day) and for electricity for reefer units (a rule of thumb is 0.025 mt/day reefer, depending on inside/outside temperature). Calculating actual fuel consumption is very complex as vessel draft, wind, waves, currents and date of the last hull scraping affect fuel consumption.

Some seaboards are under sulphur emission restrictions, limiting the percentage of sulphur content in the bunker, denoted LSFO (Low Sulphur Fuel Oil, as opposed to High Sulphur Fuel Oil (HSFO)) is supplied with a premium to the bunker price and has reduced availability.

Ports

A port may consist of several terminals competing for the cargo traffic in the corresponding port. A carrier will typically use a single terminal at every port because of connections between services. Ports have a maximum draft and the berths have a maximum length. This results in vessel-port incompatibilities for some port-vessel combinations. A container vessel is piloted in and out of port by a pilot employed at the port authorities. *Pilot times* may be several hours and for ports situated up a river bed it can be 8-10 hours. At some terminals a berthing slot is reserved in advance, whereas others serve vessels by a first-come-first-serve (FCFS) basis. A vessel may have to wait for a given pilot time or wait for an available berthing slot at FCFS ports. To enter/leave some ports a vessel may also have to wait for high tide. A port call cost is paid to enter the port, which depends on vessel specifications. The port call cost covers expenses to the pilots, tug boats, the port authorities and the terminal. Once the vessel has entered a terminal the vessel will commence unloading and loading of cargo. Cargo handling is often referred to as *moves* in general. Each move is associated with a cost to cover the expenses of the cranes, terminal

crew and terminal administration. A transshipment move will typically cost less than a load and an unload move together. The crane height and length of the terminal may also result in port-vessel incompatibilities. The number of moves per unit time, that a port is able to perform is denoted as its *productivity*. There is usually a distinction between main, transshipment, and feeder ports. Feeder ports are usually small and their productivity may vary with the level of technology deployed at the feeder port. Main ports have a large quantity of import/export and some transshipment facilities. Most main ports will have medium to high productivity. Lastly, transshipment ports such as Balboa, Singapore and Algeciras do not have extensive import/exports but serve to transship between services and inter-modal transport. Transshipment ports usually have a high productivity.

Port Stay

In real life port stay times will vary greatly due to the different productivity and strategies of terminals either having reserved berthing slots or perhaps serve vessels on a first come first serve basis. Furthermore, the port stay depends on the amount of cargo to load and unload in the given port for that particular calling. The benchmark suite does not include measures of productivity, nor do we model the port time on basis of the cargo flow, and therefore it has been chosen to fix the port time to an identical measure for all ports and vessel classes. This is motivated by the observation that larger vessels in practice will have higher productivity, since they call more efficient ports served by more cranes. This means that vessels irrespective of size often have comparable port stay times, an analysis of Maersk Line ports stay times roughly supports this.

Canals

The two large canals of Panama and Suez enable fast transport between continents. To traverse the canals, a substantial fee is charged. The canals offer significantly faster transit times and also reduced operational costs due to the reduction of the sailing distance. Some sailing passages may have draft and width restrictions such as the Panama or Kiel Canal resulting in a significant detour for larger vessels. Some, as the Bosphorus straight, are the only entrance to a sea body and the limitations of these straight thus dictate the size of entering vessels. The current restriction on vessel size of the Panama canal is very decisive in the services of the American east coast from the Far East and Australasia. No such restriction exists for the Suez canal with current container vessels.

Equipment

Containers are generally available in lengths 20, 40, and 45 feet each of which exist in different types as: Dry (DC), refrigerating (*reefers*), open top (OT) and rack, plus additional specialized containers. A customer will generally require a specific type of container, though in some cases containers are used outside their designated scope. *Dry* equipment covers roughly 80% of a carriers equipment pool (Stopford, 2009) with remainder mainly being *reefers*. There is a similar distribution on container sizes with around 80% being 40' containers (Stopford, 2009).

2.4.4 Demand and Customers

The goods and customers of containerized transports are plentiful and cover any type of manufactured goods, but can also be bulk-like cargo such as stone, waste-paper or refrigerated commodities like fish or fruits. A service will usually focus on customers in a *trade* (sometimes multiple trades), e.g., *Asia to Europe* or *South America to North America*. They may be grouped into east-west trades, which have larger volumes, allowing for economy of scale deploying huge vessels and into north-south trades, which have much refrigerated cargo requiring specialized vessels and faster transit time. Similarly all trades have special characteristics with regards to volume, service level requirements, etc. that govern how a service can compete in the trade.

Demand

The production and consumption of some commodities vary over the year, some following harvest seasons, others following holidays and festivals or summer vacations. As a result, demand varies, some on a global level others only affecting a single port or trade. The biggest of these is Christmas, which, combined with other effects, generates the yearly peak season in the third quarter, allowing warehouses to fill up prior to Christmas shopping. Another peak happens prior to Chinese New Year as Chinese factories export their goods before closing down for the festivities. A customer or shipper will usually only pay for a container transport once it is on the vessel or delivered, i.e., there is no fee for booking and no fee for not submitting a cargo for a booking. This risk-free booking policy for the shipper causes many problems for the carrier and shipper:

- It makes it hard to forecast the demand of cargo for some vessel departures, (although it can be used at ports called later), so, the carrier will overbook departures. This causes delays of cargo when forecasts and bookings fall short.
- Shippers know the overbooking as a risk, and, to counter this, they will intentionally book for more containers than they expect to ship, often spread at different carriers to optimize flexibility. This especially happens in peak seasons as the third quarter, where capacities are the most pressed.

Containerized transport has been a growing industry for many years. Between 1983 and 2006 world GDP growth was 4.8% per year, whereas container cargo growth averaged to 10% per year Stopford (2009). The trend has continued in recent years looking at the data from The World Bank with an exception of the year 2009 where the industry was suffering from the worldwide financial crisis. The long history of fast growth in containerized cargo has given an expectation of continuous growth in the industry. The delivery time of ordered vessels is several years and this makes the market slow to adapt to increased demand with resulting huge fluctuations in revenues as seen in the aftermath of the financial crisis in 2009.

Revenue

In general the revenue of a cargo is closely related to the *demand-supply* balance between the volume of cargo to be transported and the capacity offered by container carriers. But many other aspects come into play. Specialized cargo, requiring specific equipment or different administrative handling will give higher revenue, for instance refrigerated or military cargo. As the demand is not symmetrical (e.g., Asia exports much more than is imported) but the supply is symmetrical (vessels return to Asia to be reloaded), hence also the revenue is not symmetric. Transporting a container from Asia to Europe can easily cost three times more than the reverse (Alphaliner charter rates 2000-2010).

The demand fluctuations over a year influence the revenues, with the highest revenues in the third quarter, but with variations over trades.

Service level

The specific service a customer is presented with, for transporting a container from port A to port B , the *product*, can be classified by a number of parameters:

- The price: A basic rate, subject to additional fees and surcharges.
- Bunker Adjustment Factor (BAF): A variable price component dependent on the bunker price, making it possible for the carrier to share the risk of oil price volatility with the shipper.
- Transit time: The time to transport a container from A to B .
- Transshipments: The cargo is reloaded onto a new vessel in a third port. A direct product is usually preferred due to the decreased risk of missing a connection.

- **Frequency:** A weekly service calling a port is preferred to a bi-weekly service due to increased flexibility.
- **Reliability:** Ocean carriers differ in reliability of services and the products they operate (Notteboom (2006), Maersk Line).
- **Paperwork:** As documentation and administration, different carriers and products require different paperwork, adding complexity for the shipper.
- **Equipment:** The carrier must supply the customer with a container prior to transporting.
- **Weekday:** Vessels departing on a Friday can, e.g., be preferable to a departure on Monday, as it can shorten the supply chain with three days (for a factory closed in the weekend).

All these factors can be relevant for a shipper, with price and transit time often being the key factors.

Routing (Products)

As a result of trade policies there may be various *cabotage rules* within countries restricting internal transport within the country to routings on vessels flagged/owned/operated in the country. Cabotage applies to transshipments of cargo destined within the country and also for empty containers. Other special rules may exist, e.g., embargoes: when a country *A* is embargoing a country *B*, cargo from/to *B* can not be transshipped in *A*. Dangerous goods are subject to *IMO rules* with regards to the quantity and the placement of these goods on the container vessel.

Competition & Partnerships

As mentioned the economies of scale in liner shipping can generate huge savings for larger volumes. This also motivates carriers to cooperate on operating services or subcontract. Such partnerships exists in various forms.

Foreign Feeder Agreements (FEF) A service operating between a hub and a few proximate ports is called a *feeder*. These are usually operated by smaller vessels (less than 1250 FFE) and the cargo is transshipping in the hub to some ocean-going service. The feeder carrier will combine volumes from other global carriers to achieve economy of scale.

Alliances and Vessel Sharing Agreements (VSA) The significant economies of scale achieved by deploying larger vessels is the motivation for multiple carriers to enter a VSA. Two or more carriers share a round trip, making it possible to deploy larger vessels and providing higher frequency of service. Deploying large vessels presents a significant saving in operating cost for each carrier (see Figure 13.11 in Stopford (2009)) without reducing the service levels presented to their customers. In practice, VSAs are complicated because the partners seldom have matching services and demand, available vessels, etc., but VSAs are widely used and a central part of liner shipping. For more details on the mechanics of VSAs, refer to (Agarwal and Ergun, 2008b) for game theoretic observations on the subject.

Slot Charter Agreements (SCA) are a combination of FEF's and VSA's. One carrier will enter a contract to use capacity (*slots*) on another carrier's service. This can either be on the full round trip of the service, allowing for the slot to be used differently in head and back haul, or on a specified part of the rotation. The contract is usually for a fixed number of slots which is paid for *used/unused*, sometimes with an option for extra slots as *pay per use*.

2.5 The benchmark suite for LSNDP

The purpose of the benchmark suite is to provide data for prototype implementations of models and methods for the liner shipping network design and fleet deployment problem at a strategic level. The literature review displays three important factors for the development of the benchmark suite:

1. The research community views the problem from different angles in terms of constraints and costs. As a consequence the data must address the important factors seen across the entire research field and also incorporate data on foreseeable extensions to current models.
2. A “base” model is needed in order to compare methods.
3. Currently, exact methods only solve very small instances, while heuristics such as the tabu search by Alvarez (2009) caters for a large instance. As a consequence the benchmark suite should contain small, realistic instances, that may be solved to optimality within foreseeable future and large instances for heuristic development.

There are at least two pressing research questions on the LSNDP: efficient modeling of bunker consumption as a cubic function of speed and incorporating the level of service. Global carriers such as Maersk Line put great effort into creating a network design of low bunker consumption to meet a demand for environmentally friendly transportation of goods and improve the economy of the network. As noted by Notteboom and Vernimmen (2009), the price of bunker has a significant impact on the network design and the fleet deployed. Therefore, formulas for bunker consumption based on vessel class are included in the benchmark suite to allow for calculations of the overall bunker consumption as a function of speed. At the same time, Notteboom and Vernimmen (2009) state that customers are demanding a high frequency and low transit times and therefore, level of service and competitive transit times are important factors in LSNDP. The benchmark suite includes a set of maximal transit times on commodities as we believe level of service and transit time incorporation to be important open research questions in the liner service network design problem.

We have chosen to provide vessel data in terms of six fictitious, but realistic vessel classes, where multiple vessels share similar characteristics. We believe a global carrier has several similar vessels assigned to a service (Agarwal and Ergun, 2008a; Notteboom and Vernimmen, 2009; Alvarez, 2009). A prototype implementation will contain the most important details, but omit details believed to be non-decisive for the conceptual understanding of the problem. Therefore, the benchmark suite does not contain different equipment types such as reefer and high cube containers. The weight of vessels and cargo in particular are omitted from the benchmark instances.

Assigning specific vessels to a specific service is an operational issue dependent on cabotage rules. Hence, the benchmark instance do not indicate the country of registration of vessels in the present data set. Likewise, IMO rules apply to specific cargoes and stowage plans, which is out of scope of the considered LSNDP.

Weather and currents may have a large influence on travel times and energy consumption in a LSNDP, but we have chosen to ignore these factors to avoid further complexity.

The benchmark instances are not meant for actual scheduling of services. Therefore tidal information on ports is not provided although they may be decisive for a specific berthing window at a port. On the same note, the specific weekday on which a demand becomes available for transport out of a given port is not included in the data of the benchmark suite.

The following section discusses data objects and data generation. A discussion on the range of instances needed to qualify the benchmark suite follows.

The benchmark suite originates from publicly available data and past data for Maersk Line.

External data sources are: “Distances between ports - Pub. 151”, National Imagery and Mapping Agency, The Hamburg Index, Vereinigung Hamburger Schiffsmakler und Schiffsbagenten, “Charter Rates 2000-2010”, Alphaliner charter rates 2000-2010, “Container Freight Rate Insight”, Drewry Shipping Consultants.

The data from Maersk Line is from a given anonymous year and has been subject to reasonable random perturbation and measures to anonymize the data in order to protect the sensitivity and confidentiality of Maersk Line.

The purpose of the benchmark suite is to qualify algorithms for scalability, quality, and robustness by providing realistic data for both the scale, structure and complexity of the liner shipping network design problem. We have chosen not to anonymize the actual ports to be able to evaluate the network from a business perspective, but the data still does not represent an actual business case, nor does it contain all relevant data. We hope the benchmark suite can be used to perform strategic analysis on various scenarios. The following section describes the data generation process in terms of the origin of the data and the logic used to derive the benchmark suite.

2.5.1 Data objects and data generation

An instance consists of the following data: **Port list** with port call cost as a function of vessel capacity (explained in more details in Section 2.5.2). **Fleet list** with design speed and bunker consumption at design speed and idling (Section 2.5.3). **Distances** contains an all-to-all list of travel distances (Section 2.5.4), and **Demands** specifies a set of origin-destination pairs and their corresponding demands (Section 2.5.5). Finally, the **Graph** file defines a graph representation of the considered problem with way points. The data items are illustrated in Figure 2.5 giving an overview of the attributes of each item.

<table><tr><th>Port List P</th></tr><tr><td>Port α^P</td></tr><tr><td>Name n^P</td></tr><tr><td>Country o^P</td></tr><tr><td>Cabotage a^P</td></tr><tr><td>Region r^P</td></tr><tr><td>Latitude x^P</td></tr><tr><td>Longitude y^P</td></tr><tr><td>Max berthing length ℓ^P</td></tr><tr><td>Draft δ^P</td></tr><tr><td>Move cost m^P</td></tr><tr><td>Transshipment cost t^P</td></tr><tr><td>Fixed portcall cost f^P</td></tr><tr><td>Variable portcall cost v^P</td></tr></table>	Port List P	Port α^P	Name n^P	Country o^P	Cabotage a^P	Region r^P	Latitude x^P	Longitude y^P	Max berthing length ℓ^P	Draft δ^P	Move cost m^P	Transshipment cost t^P	Fixed portcall cost f^P	Variable portcall cost v^P	<table><tr><th>Fleet List F</th></tr><tr><td>Vessel class v^F</td></tr><tr><td>Capacity c^F</td></tr><tr><td>TC rate T^F</td></tr><tr><td>Draft δ^F</td></tr><tr><td>Min speed s_{\min}^F</td></tr><tr><td>Max speed s_{\max}^F</td></tr><tr><td>Design speed s_*^F</td></tr><tr><td>Fuel consumption f_*^F</td></tr><tr><td>Fuel consumption f_0^F</td></tr><tr><td>Quantity of vessels q^F</td></tr><tr><td>Suez fee s^F</td></tr><tr><td>Panama fee p^F</td></tr></table>	Fleet List F	Vessel class v^F	Capacity c^F	TC rate T^F	Draft δ^F	Min speed s_{\min}^F	Max speed s_{\max}^F	Design speed s_*^F	Fuel consumption f_*^F	Fuel consumption f_0^F	Quantity of vessels q^F	Suez fee s^F	Panama fee p^F
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<table><tr><th>Cargo Demand List C</th></tr><tr><td>Origin port α^C</td></tr><tr><td>Destination port β^C</td></tr><tr><td>Quantity q^C</td></tr><tr><td>Freight rate f^C</td></tr><tr><td>Max transit time t^C</td></tr></table>	Cargo Demand List C	Origin port α^C	Destination port β^C	Quantity q^C	Freight rate f^C	Max transit time t^C	<table><tr><th>Distance List D</th></tr><tr><td>Origin port α^D</td></tr><tr><td>Destination port β^D</td></tr><tr><td>Draft δ^D</td></tr><tr><td>Distance d^D</td></tr><tr><td>Suez traversal s^D</td></tr><tr><td>Panama traversal p^D</td></tr></table>	Distance List D	Origin port α^D	Destination port β^D	Draft δ^D	Distance d^D	Suez traversal s^D	Panama traversal p^D														
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Panama traversal p^D																												
<table><tr><th>Graph</th></tr><tr><td>Vertex (Port, Waypoint, Canal)</td></tr><tr><td>Edge (Source, Target, Distance, Draft)</td></tr></table>	Graph	Vertex (Port, Waypoint, Canal)	Edge (Source, Target, Distance, Draft)																									
Graph																												
Vertex (Port, Waypoint, Canal)																												
Edge (Source, Target, Distance, Draft)																												

Figure 2.5: Data objects of an instance

The benchmark suite does not dictate a planning horizon for the models at hand. However, a weekly frequency is a business standard (Notteboom and Vernimmen, 2009; Network and Product at Maersk Line) and hence demand scenarios are based on weekly calls. A week is easily scaled to longer planning periods if desired. The cost and revenues are not meant to reflect the actual cost of any carrier. The costs and revenues are constructed with care to reflect the relative cost

structure within the network. This means that a large port is proportionally cheaper to call at than a small port. Likewise, it is cheaper to transship a container in Asia than in Europe or the US. Revenues reflect demand and supply such that visiting a distant port with low demand require higher revenues per FFE than a central port with extensive own demand and supply.

2.5.2 Port List

Ports are identified by UNLOCODE, a unique 5-character identifier (for Europe)). The port list specifies the ports in the instance and contains the following fields: Port (in UNLOCODE); Name; Country; Continent; Cabotage region if applicable; Revenue Region (maps port to the revenue data supplied by Drewry Shipping Consultants); Latitude; Longitude; Maximal berthing length (in meters); Maximal acceptable vessel draft (in meters); (Un)Load cost per FFE (in USD); Transshipment cost per FFE (in USD); Fixed port call cost (in USD); Variable port call cost per FFE (in USD) as seen in table 2.2.

The port list reflects real ports ensuring that a solution can be mapped to a geographical coverage. This allows network planners and others without optimization background to evaluate a proposed network.

Port α^P	Name n^P	Country o^P	Cabotage a^P	Region r^P	Long x^P	Lat y^P	Draft δ^P	Move m^P	Trans t^P	Fixed f^P	Var v^P
CNYTIN	Shenzhen	China	China	South China	114.26	22.58	13.5	176.69	78.14	7220.21	3.8

Table 2.2: Example of port entry. Port is given by UNLOCODE. **Draft** is the maximal draft. **Move** is the (un)load cost and **Trans** is the cost of a full transfer for transshipment. **Fixed** is the fixed port call cost and **Var** is the variable port call cost as a function of the capacity.

The transshipment cost will usually be lower than the sum of loading and discharging as transshipping does not require customs paperwork at the terminal in question. In an optimization model, the load and discharge may be viewed as fixed costs once the demand is chosen for trans-port.

The geographical location and the port size will determine a fixed cost for visiting the port and a variable cost related to the capacity of the vessel visiting the port. This relation is deduced experimentally from the perturbed Maersk Line data giving a non-zero fixed cost and a variable cost corresponding approximately to a linear function of the capacity.

2.5.3 Fleet List

Six generalized vessel classes are constructed from the fleet list of Maersk Line (A.P. Møller-Mærsk Fleet List) representing realistic capacity classes in the network. A vessel class will contain the following information: TC rate per day per vessel (in USD), this value includes OPEX, crew and maintenance costs; Capacity (in FFE); Design draft (in meters); Minimal speed (in knots); Maximal speed (in knots); Design speed (in knots); Daily bunker consumption in metric tonnes at design speed; Own consumption in metric tonnes when idling at ports; Number of vessels in the vessel class currently in the fleet; Suez canal fee (in USD); Panama canal fee (in USD) as seen in Table 2.3.

The minimal- and maximal-speeds referred to in the fleet list are not the technical minimum/maximum speeds. The speed interval is related to the average speed on a service deploying the vessel class in question. This means the vessel will be doing less than the minimal speed on part of the service and hence the minimal speed referred to in the fleet list is higher than the technical minimum speed. The technical maximum speed will also be higher than the maximum speed referred to in the fleet list and likewise refers to the maximal average speed that can be assigned to a service with the vessel class in question deployed. The minimal speed is the “slow steaming” speed of a service deploying the vessel class. The only size measure describing the vessels is draft. Other measures such as length and beam width, should be seen as a proportional function of draft.

This generalization is done to simplify the problem, keeping in mind that a full description of the problem can extend to more than this one size measure without losing applicability.

Vessel class v^V	Cap c^V	TC/\$ T^V	Draft δ^V	Min s s_{\min}^V	Max s s_{\max}^V	Des s v_*^V	Fuel* f_*^V	Fuel0 f_0^V	Quantity q^V	Suez/\$ s^V	Panama/\$ p^V
Feeder_450	450	5000	8.0	10	14	12.0	18.8	2.4	38	175769	64800
Feeder_800	800	8000	9.5	10	17	14.0	23.7	2.5	77	218445	115200
Panamax_1200	1200	11000	12.0	12	19	18.0	52.5	4.0	124	267217	172800
Panamax_2400	2400	21000	11.0	12	22	16.0	57.4	5.3	161	413533	345600
Post_panamax	4200	35000	12.5	12	23	16.5	82.2	7.4	91	633007	
Super_panamax	7500	55000	13.5	12	22	17.0	126.9	10.0	10	1035376	

Table 2.3: Example of the fleet list. **Vessel class** is the name of the class. **Cap** is the capacity in FFE. **TC** is the TC rate. **Draft** is a single measure expressing width, length and draft for port/vessel compatibility. **Min s**, **Max s** is the minimum and maximum average speed interval of the vessel whereas **Des s** is the design speed. **Fuel***, **Fuel0** are bunker consumption at design speed and idling respectively. **Quantity** is the available number of vessels in the fleet with these specifications. **Suez**, **Panama** are the canal fees for the respective vessel classes.

The fleet list reflects the demand scenario of the instance as a trade lane has a smaller fleet than a world instance.

TC rates

The TC rates fluctuate with the market and hence capturing TC rate is very dependent on the date it was retrieved. We have compared data from Alphaliner charter rates 2000-2010 and “HAMBURG INDEX Containership Time-Charter-Rates” Vereinigung Hamburger Schiffsmakler und Schiffsagenten, which have charter rates for vessels up to 4000 and 4800 TEU respectively. For vessel classes above 4800 TEU, we have constructed a TC rate based on the percentage increase of TC rate in the Maersk Line data to corresponding capacity intervals of the fictitious vessel classes and have increased the Hamburg index of charter prices with the corresponding percentage. The TC rates have been ceiled to the nearest thousand for vessel classes below post panamax size and to the nearest 5000 for the two largest vessel classes.

Bunker consumption

Bunker cost is a significant cost component of the network. The bunker curves for the generalized vessel classes are based upon the formulas of Stopford (2009) and Alderton (2005). There is general agreement that bunker consumption, F , may be estimated by a cubic function

$$F(s) = (s/s_*^V)^3 \cdot f_*^V \quad (2.1)$$

for any speed s between the *min speed* s_{\min}^V and *max speed* s_{\max}^V of the vessel V , where s_*^V is the design speed, and f_*^V is the fuel consumption at design speed. This function disregards draft although it is a significant factor, so the bunker consumption should be the consumption at design draft as well. The design draft is the draft the vessel has with an average load and for which it is designed to have the lowest bunker consumption as a function of speed. The fleet list specifies the design speed and consumption at design speed for the cubic formula for each vessel class. The bunker price is variable and the effect of bunker price could be a scenario for a user of the benchmark suite. In the computational results we use a flat bunker price of 600 USD per ton.

2.5.4 Distances

The distances between ports are based on information from the National Imagery and Mapping Agency (NIMA, 2001). The data from NIMA contain distances from each port to major way points (ocean junctions) and ports in the vicinity. The data enables a mapping of global sailing

routes onto a graph of ports and way points. Each arc is given specifications such as the maximal draft and maximal width such as the Panama canal, which has a width restriction. A single-source shortest path algorithm between each pair of ports determines the shortest feasible path for each vessel class based on this data. It means that the distance between Oakland on the US West Coast and Savannah on the US East Coast will differ for vessels able to traverse the Panama canal as opposed to vessels not able to traverse the Panama canal. To account for canal cost, a parameter indicates whether the distance is based on a visit to the Suez Canal and/or the Panama canal. The distance file is a table with multiple entries for each port pair depending on draft limitations and canal usage.

The distance data consists of: Origin port α (in UNLOCODE); Destination port β (in UNLOCODE); Distance (in nautical miles); Draft limit (in meters) δ^D ; Suez traversal (yes/no); Panama traversal (yes/no) as seen in Table 2.4.

Origin α^D	Destination β^D	Distance d^D	Draft limit δ^D	Suez traversal s^D	Panama traversal p^D
GBABD	USOAK	7953	12	No	Yes
GBABD	USOAK	13862		No	No

Table 2.4: Example of distance file. **Origin** is the origin (in UNLOCODE), **Destination** the destination (in UNLOCODE). **Draft limit** is the draft limit in meters (if relevant). **Suez traversal**, **Panama traversal** indicate whether the respective canals are passed to calculate canal fees.

The canal dues for the world’s two most important canals; Panama and Suez, are included in the benchmark suite. These are based on their published cost structure (Suez; Panama Canal) and created for the relevant vessels classes. Additional canals are omitted from the benchmark suite.

After manual inspection of the distances, several entries were identified to be erroneous. We have attempted to verify the correctness of several distances through external data sources and to correct obvious errors in the data. However, the distances are not guaranteed to be the shortest distances due to the selection of way points and due to possible undetected errors.

The distances have been collected and put into a graph $G = (V, E)$. The set of ports P and the set of way points W constitute $V = P \cup W$. The edge set E is from National Imagery and Mapping Agency. Two distance files are generated

1. An all-to-all distance matrix only for the port set $P \times P$. This graph has few vertexes and many edges.
2. A (sparse) graph $G = (V, E)$, of the instance with $V = P \cup W$ nodes.

The graph file contains all vertices’s and edges needed to navigate between any pair of ports in P with the fleet F . The files are generated using a shortest path algorithm between all pairs of ports for each vessel class in the fleet F .

2.5.5 Cargo Demands

Realistic demand data that captures the asymmetry in world trade is important when deciding on capacity and port sequences. A maximal transit time for each demand is provided for future models incorporating level of service or maximal transit time constraints.

The demand table contains the following information: UNLOCODE of the origin port α ; UNLOCODE of the destination port β ; Quantity (in FFE); Freight Rate (in USD per FFE); Maximal transit time (in days) as seen in Table 2.5. The demands are assumed to be the expected weekly demand.

For world instances, demands are aggregated onto the main ports.

The revenues are based on the trade lane prices found in “the container freight rate index”, courtesy of Drewry Shipping Consultants. The rates are independent of any carrier and contains

Origin α^C	Destination β^C	Quantity q^C	Freight rate f^C	Max. transit time t^C
USLSA	CNYAT	370	1500	15

Table 2.5: Example of demand file. **Origin**, **Destination** give the origin and destination of the demand. **Quantity** states the number of FFE needed. **Freight rate** is the freight rate in USD per FFE. **Max transit time** states the maximum transit time in days.

reliable market data collected across several carriers and markets. Drewry Shipping Consultants do not keep data for South America West coast and some region pairs. In such cases the official rates from the website of Hapag-Lloyd have been applied. The revenues are taken from a specific period of time, and it should be noted that these fluctuate greatly dependent on volume changes, new buildings, bunker price, etc. The revenue is 70% of the market freight rates as we are only dealing with the network cost and must be able to pay for the administrative overhead estimated to 30% by Stopford (2009). The volume of the origin/destination ports along with the distance to the nearest main or transshipment port are used to vary the trade lane price according to the demand structure, such that a demand from a transshipment/main port is cheaper than a demand from a feeder port and feeder port revenues are dependent on the distance from the main port. Lastly, the freight rates are perturbed within an interval of $\pm 5\%$ to reflect price variance of different markets and cargoes.

2.5.6 Instance range of the benchmark suite

The instances are meant to challenge both exact and heuristic methods. Exact methods are currently limited to approximately 10 ports, whereas the tabu search heuristic of Alvarez (2009) provides results for 120 ports. Furthermore, the instances are meant to encourage the progress of method development and hence include instances of real life scale, although we cannot yet solve them to optimality. Most of the instances are subsets of a larger network, where the subset is selected to highlight various aspects of the LSNDP such as using a single hub or multiple hubs, the importance of draft restrictions or the capacity in terms of the fleet size is low compared to the demand and vice versa. The provided instances are listed in Table 2.6. In addition to the instances we have generated a set of rules to transform the *base* cases into two additional scenarios, where there is *low capacity* and *high capacity* compared to demand volume. The scenarios are created by adjusting the fleet data in terms of the number of vessels available and the TC rate, which is adjusted to reflect whether there is a deficit or surplus of capacity in the market. For the *low capacity* case TC-rates are multiplied by 1.4 to reflect a 40% increase of TC-rates, while the fleet quantity is decreased by 20% by multiplying the quantity with 0.8 and rounding to nearest integer value. In the *high capacity* case TC-rates are decreased by 20% by multiplying TC-rate by 0.8 and the fleet quantities are increased by 20% multiplying each vessel class quantity by 1.2 and rounding it to the nearest integer. The instances may be found at <http://www.or.man.dtu.dk/English/research/instances> under the name *LINER-LIB 2012*.

The Pacific and the single/multiple hub instances should be challenging for the exact approaches and will also be applicable to model development as models of LSNDP can be verified by analyzing the results of the benchmark suite.

To have a medium sized multi hub instance, the Mediterranean case has been constructed. It includes 39 ports between Morocco and Suez and includes ports in the black sea. Several ports have large demand sets throughout the seaboard. The demands in the Mediterranean instances are constructed and not comparable to a real world setting. The AsiaEurope trade lane instance and World instances should be challenging for the heuristics developing new methods, neighborhoods and competing for the current best known objective value. Exact methods solving these instances to optimality are not expected in the near future. We do not provide computational results for the WorldLarge instance as the size is currently considered out of reach even for heuristic methods.

The Benchmark suite *LINER-LIB 2012* can also be used in other optimization models related

Category	Instance and description	Ports	Dmnds
Single hub instances	Baltic Baltic sea with Bremerhaven as hub	12	22
	WAF West Africa with Algeciras as hub to West African ports	19	38
Multi hub instances	Mediterranean Mediterranean with Algeciras, Tangier, Giao Tauro as hub	39	369
Trade lane instances	Pacific (Asia-US West)	45	722
	AsiaEurope Europe, Middle East and Far east regions	111	4000
World instances	Small 47 Main ports worldwide identified by Maersk Line	47	1764
	Large 197 ports - the majority of ports serviced directly by Maersk Line	197	9630

Table 2.6: The instances of the benchmark suite with indication of the number of ports and the number of distinct origin-destination pairs. The instances may be found at <http://www.or.man.dtu.dk/English/research/instances>

to liner shipping network design such as evaluation of a set of candidate services, synchronization of services to reduce transit times, sailing speed optimization etc.

2.6 Reference Model for the LSNDP

The following model is based on the model presented in Alvarez (2009) with extensions to handle butterfly rotations and weekly frequencies. Butterfly rotations (illustrated in Figure 2.6) make it more difficult to account for transshipment costs. The planning horizon remains open, but in the route generation a constraint has been added such that the number of vessels are aligned with the planning horizon and service duration to provide a frequency of service within a limited band of a (bi)-weekly frequency.

2.6.1 MIP formulation

In this section, we present a possible formulation of the LSNDP. In the maritime sector, a service is defined as a sequence of ports to be visited following a published schedule. While we adhere to this convention, the core entity of our model must be more specific in order to capture all transshipment costs properly. We therefore define a *rotation* as a particular configuration of a service, vessel class, number of vessels deployed, and speed.

Our formulation of LSNDP addresses butterfly routes, as these are often used in practice. This means that the flow balance equations need to be extended. In order to balance the flow at butterfly nodes correctly, a three-index formulation is necessary. Consider the rotation in Figure 2.6, where port c is the butterfly port. With a traditional flow balance at port c , cargo that arrives from b might be redirected towards port a under the same conditions as cargo that continues on to port d . However, cargo travelling on the port sequence $b-c-a$ must be unloaded at c as the vessel continues to d , and later reloaded as the vessel arrives from e . Hence, it appears that we must track the latest port visited by the cargo, as well as its immediate destination, which we can encapsulate using a three-index formulation. Notice, that the cardinality of the problem is not affected by this representation, given that one requires the same number of port pairs or port triples to specify a service uniquely.

The following sets are used in the formulation (In square brackets it is described how the sets relate to the graph defined in Section 2.5.4 and data objects defined in Figure 2.5):

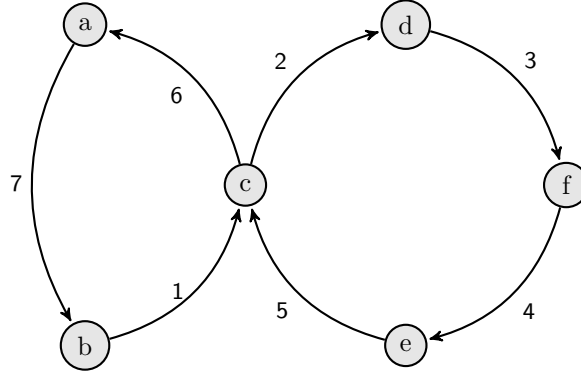


Figure 2.6: An illustration of the flow balancing of butterfly routes. The numbers on the arcs denotes the sequence the vessel sails in the butterfly. Commodities travelling from b to a either follows the vessel on route $b - c - d - f - e - c - a$ or are transshipped at c , where the handling cost of the container must be paid in full.

R	All rotations in the model, indexed using r .
P	All ports in the model. [$\mathbf{P} := P$]
E	Set of all possible edges in the model [$\mathbf{E} \subset P \times P$]. All edges are directed and uncapacitated.
\mathbf{E}_r	Set of edges used in rotation r .
Ω_r	Set of ordered port triples (h, i, j) in rotation r . The triples are ordered in the same manner as they will be visited by a vessel in the rotation.
V	Vessel classes in the model, indexed using v . [$\mathbf{V} := F$]
G	Set of port pairs with demand for transport, indexed using (o, d) . [$\mathbf{G} \subset P \times P$].

The following parameters represent the known data for the problem (In square brackets it is described how the parameters are calculated from the benchmark suite data):

a_{ij}^v	Canal costs for vessels of type v when traversing arc (i, j) . [s^V (Suez), p^V (Panama)]
c^v	Capacity (in FFE) of vessels of type v . [c^F]
d_j^v	Port call costs for vessels of type v when entering port j . [$f^P + v^P \times c^v$, where c^v is the number of FFEs the vessel can carry].
e	Fuel price per ton. [\$600 for all instances]
f^v	Daily running costs for vessels of type v over the entire planning horizon. [$f^v = T^v \cdot 180$ as we have a 180 days planning horizon, and daily TC rate is T^v]
\tilde{f}^v	Cost or revenue of excluding a vessel of type v from operations. This can be the revenue obtained from chartering the vessel out over the entire planning horizon, or the cost of laying up the vessel, depending on transportation market conditions. [f^v (vessel in use), $f^v - \tilde{f}^v = 0$ (vessel not in use)]
g^{vs}	Fuel consumption (tons per mile) for vessels of type v steaming at speed s . [A table of possible values of speed and g^{vs} is generated a-priori by the program using equation (2.1)]
h^v	Fuel consumption (tons per day) for vessels of type v when idle at port. [f_0^F]
k_{od}	Total demand (in FFE) to the liner company for transport from o to d over the planning period. [q^C where $o = \alpha^C$ and $d = \beta^C$]
l_{ij}^v	Length in nautical miles of a direct sailing from port i to port j using vessel type v . Notice, for instance, that canal width restrictions will result in different voyage lengths for wider vessels. [d^D where $i = \alpha^D$, $j = \beta^D$ and $\delta^D > \delta^F$]
m_r	Number of round trips performed by a vessel on rotation r during the planning period. We do not impose an integrality restriction on this parameter, as we assume that a fractional portion of the voyage can be completed at the beginning of the following planning period.
p_j^v	Time at port j for vessels of type v . [Based on historical data we set $p_j^v = 24h$ regardless of vessel size]
q_{od}	Revenue from transport of one FFE from o to d . [f^C where $o = \alpha^C$ and $d = \beta^C$]
\tilde{q}_{od}	Penalty for failing to transport one FFE from o to d . [\$1000 USD per FEU in all cases]
s_r	Speed of all vessels used in rotation r . [generated by the algorithm. The code tries several speed-vessel-route combinations (respecting the vessel speed range) and picks the one that leads to a better objective value for that iteration]
u_j	Cost of lifting one FFE at port j . [m^P]
t_j	Cost of transshipping one FFE at port j . [t^P]
v_r	Vessel type used in rotation r . [v^F]
z^v	Number of vessels of type v available. [q^F]

Finally, the decision variables used in the MIP are:

$X_{(ij)d}^r$	Number of containers travelling to their final destination at port d along edge (i, j) on rotation r .
$U_{(hi)d}^{rs}$	Number of containers travelling to port d which arrive to port i via edge (h, i) of rotation r for transshipment to rotation s .
$W_{(id)}^r$	Number of containers travelling to port d reaching their final destination via edge (i, d) using rotation r .
V_{od}^r	Demand from port o to port d that enters the network for the first time as it is loaded to a vessel in rotation r .
O_{od}	Demand from port o to port d that will not be serviced by the liner company.
Y^r	Number of vessels assigned to rotation r .

The MIP model is presented below, followed by an explanation of the objective function and all the constraints.

$$(VRD) \quad \text{minimize} \quad Z_{MP} = \sum_{r \in \mathbf{R}} Y^r f^{v_r} + \sum_{v \in \mathbf{V}} \tilde{f}^v \left(z^v - \sum_{r \in \mathbf{R}: v_r = v} Y^r \right) + \quad (2.2a)$$

$$\sum_{r \in \mathbf{R}} Y^r m_r \sum_{(i,j) \in \mathbf{E}_r} (eh^{v_r} p_j^{v_r} + eg^{v_r s_r} l_{ij}^{v_r} + d_j^{v_r} + a_{ij}^{v_r}) + \quad (2.2b)$$

$$\sum_{(o,d) \in \mathbf{G}} \left(\tilde{q}_{od} O_{od} - q_{od} \sum_{r \in \mathbf{R}} V_{od}^r \right) + \quad (2.2c)$$

$$\sum_{r \in \mathbf{R}} \sum_{(h,i,j) \in \Omega_r} \left(u_i \left(W_{(hi)}^r + \sum_{\substack{d \in \mathbf{P} \\ d \neq i}} V_{id}^r \right) + t_i \sum_{\substack{d \in \mathbf{P} \\ d \neq i}} \sum_{\substack{s \in \mathbf{R} \\ s \neq r}} U_{(hi)d}^{rs} \right) \quad (2.2d)$$

subject to

$$X_{(hi)d}^r + V_{id}^r + \sum_{\substack{s \in \mathbf{R}: \\ (k,i,l) \in \Omega_s \\ s \neq r}} U_{(ki)d}^{sr} = X_{(ij)d}^r + \sum_{\substack{s \in \mathbf{R} \\ s \neq r}} U_{(hi)d}^{rs} \quad r \in \mathbf{R}, (h,i,j) \in \Omega_r, d \in \mathbf{P}, i \neq dh \neq d \quad (2.3)$$

$$X_{(ij)j}^r = W_{(ij)}^r \quad (i,j) \in \mathbf{E}_r \quad r \in \mathbf{R} \quad (2.4)$$

$$O_{od} + \sum_{r \in \mathbf{R}} V_{od}^r = k_{od} \quad o, d \in \mathbf{G} \quad (2.5)$$

$$\sum_{d \in \mathbf{P}} X_{ijd}^r \leq c^{v_r} \cdot Y^r \cdot m_r \quad r \in \mathbf{R} \quad (i,j) \in \mathbf{E}_r \quad (2.6)$$

$$\sum_{r \in \mathbf{R}: v_r = v} Y^r \leq z^v \quad v \in \mathbf{V} \quad (2.7)$$

$$X_{(ij)d}^r, U_{(ij)d}^{rs}, W_{id}^r \in \mathbb{R}^+ \quad r, s \in \mathbf{R}, r \neq s \quad (i,j) \in \mathbf{E}_r \quad d \in \mathbf{P} \quad (2.8)$$

$$O_{od}, V_{od}^r \in \mathbb{R}^+ \quad r \in \mathbf{R} \quad o, d \in \mathbf{G} \quad (2.9)$$

$$Y^r \in \mathbb{Z}^+ \quad r \in \mathbf{R} \quad (2.10)$$

Expression (2.2a) in the objective function captures the daily running costs incurred by vessels in operation, as well as costs or revenues obtained when excluding some vessels from the operating fleet. The next term (2.2b) obtains the fuel costs (including consumption during steaming and while idling at ports), canal cost, and port calling fees for all active vessels. The term (2.2c) obtains the total revenues from transporting cargo as requested, and the penalties incurred when rejecting carriage requests. Finally, the term (2.2d) computes charges from loading and unloading containers, both at their origin and destination, and at transshipment points.

Constraints (2.3) balance the flow of containers loaded and unloaded from each rotation at nodes other than their final destination. Every container not destined for the port in question must either continue along in the same rotation or be unloaded for transshipment to a different rotation. Constraints (2.4) represent the flow of containers that were accepted for carriage arriving at their final destination. Every container arriving at its final destination is unloaded and grounded.

Constraints (2.5) tally demand from the originating hinterland. The variables O_{od} allow us to obtain a feasible solution for any fleet configuration. When there is not a sufficient number of vessels to transport all the demand, or when it is not economically convenient to transport all containers, the variables O_{od} are set to a positive value, indicating that some demand has been forfeited.

Constraints (2.6) impose restrictions on the total number of containers that can be transported by each edge in a rotation. The total that can be transported is limited by the number of vessels assigned to the rotation, the capacity of those vessels, and the number of trips the vessels perform over the planning horizon. Constraints (2.7) ensure that the number of vessels that are deployed

does not exceed the number of available vessels of each type. Finally, constraints (2.8), (2.9), and (2.10) impose non-negativity and integrality restrictions on the corresponding variables.

Summary of simplifications in the model

In this section we summarize some of the practical constraints and business rules stated in Section 2.4 that are *not* considered in the presented reference model for LSNBP. A subset of these constraints are accounted for in the data of the benchmark suite, while others are considered out of scope for mathematical models on the LSNBP. The reference model does not consider a maximal *Transit time* for each commodity, and transit times are supplied for each commodity in the benchmark suite for future model development. *Non-linear handling costs* in ports is not considered in the reference model or the benchmark suite data. The *port productivity* and *pilot times* for berthing are not considered in the fixed port stay of the reference model. The reference model considers a single container type and thus reefer capacity and the *bunker consumption of reefer containers* are not included in the reference model. Lastly, we do not consider *equipment cost* and *cabotage rules*.

2.6.2 Metaheuristic

The heuristic that coordinates the overall algorithm is based on a heuristic column generation framework (not to confuse with column generation Desrosiers and Lübbecke (2005) in LP) where an auxiliary problem is used to generate new rotations (columns) to the overall model (2.2)-(2.10) and a tabu search is applied to select the most promising set of rotations to constitute the network. At any iteration i , we solve a multicommodity flow problem (MCFP) with aggregated origins using the then-current set of rotations, \mathbf{R}^i . The MCFP is defined in the $X_{(ij)d}^r$ variables for the edges of the rotation, the $U_{(hi)d}^{rs}$ variables to account for transshipments between rotations with edge cost t_i and $W_{(id)}^r, V_{id}^r$ variables for (un)load to ports at their origin or destination with edge costs u_i . Rejected demand (O_{od}) is modeled by adding an auxiliary edge (o, d) with cost \tilde{q}_{od} for each commodity.

The overall framework for the heuristic column generation scheme using a MIP neighbourhood to generate rotations is similar to the method applied in Alvarez (2009). The algorithm presented in this paper differs because the MIP neighbourhood applied is new and includes more heuristic measures on cargo composition. The MIP used for route generation includes (bi)-weekly frequency constraints and respects business rules 1 and 2 from Section 2.4.2. Subtour elimination constraints are also used to generate routes of high quality during the last phase of the search.

The traffic demand that is not assigned to any of the routes in \mathbf{R}^i is the residual traffic demand for the following iteration, \hat{k}^{i+1} . For $i = 0$ there are no rotations in \mathbf{R}^i , and $\hat{k}_{od}^0 = k_{od}$.

Let us assume that, for each vessel family v , a number n_v of vessels are available at the end of iteration i . For each family of vessels where $n_v > 0$, we launch a collection of sub-problems $\text{AUX}(v, s, \kappa)$, for $s \in [s_{min}^F, s_{min}^F + 1, \dots, s_{max}^F]$ and various values of the number of vessels deployed, κ . In practice, this might give rise to a fairly large number of sub-problems. Therefore, during the initial iterations, we focus on a narrow subset of speeds and number of vessels to be deployed.

In solving the sub-problems $\text{AUX}(v, s, \kappa)$, we will typically obtain multiple feasible integer solutions. All these solutions are added to the overall model (2.2)-(2.10), since in any case the sub-problem contains some approximations, and the optimal solution to the $\text{AUX}(v, s, \kappa)$ may not be the solution that advances the overall heuristic the most. Our current implementation parallelizes the sub-problems, and we obtain batches of sixteen sub-problems that run simultaneously.

After the heuristic column generation for iteration i has terminated, we proceed to evaluate each new column ζ_j in conjunction with \mathbf{R}^i . We set $\mathbf{R}_j^{i+1} = \mathbf{R}^i \cup \{\zeta_j\}$ and run the multicommodity flow algorithm on this network. We select from amongst the \mathbf{R}_j^{i+1} the network with the best objective function, and set \mathbf{R}^{i+1} to be this network.

At some iterations, none of the candidate networks will result in a globally improved objective function. We permit the algorithm to exploit this neighborhood for a limited number of iterations.

If the overall objective function does not improve after a predefined number of iterations, we trigger a backtracking process. First, we return to the best known configuration \mathbf{R}^* . Then we delete rotations from \mathbf{R}^* based on three criteria: empty FFE-miles, underutilized legs, and mixed integer programming gap when the column was generated. The number of rotations that are deleted depends on how many times we have returned to this same \mathbf{R}^* – a larger number of returns to \mathbf{R}^* possibly indicates that this is no longer a promising point of departure, and the algorithm will remove a larger subset of rotations from it. The set of routes that results after backtracking is used as the basis for the following iteration.

If at any iteration the heuristic column generation procedure fails to produce any new columns (for instance, because no vessels are available) we delete a small number of rotations from \mathbf{R}^i . In order to reduce cycling, we add recently added rotations to the tabu list to avoid it being deleted from \mathbf{R}^i .

2.6.3 Route Generation

The auxiliary problem $AUX(v, s, \kappa)$ generates rotations based on the current network. We allow one butterfly node per rotation. We also designate one port as the "master" port of the service for ease of writing the subtour elimination constraints.

The following additional parameters are required for the formulation of the auxiliary problem.

\hat{k}_{od}	Residual demand (in FFE) to the liner company for transport from o to d . At any iteration in the heuristic, the residual demand is computed by taking the original demand, and subtracting the flow that is carried by existing rotations.
T	Length of the planning horizon, in days.
δ	Empirical parameter, estimates the amount of additional flow flowing through a butterfly node, as compared to a regular node.
$\phi_n^{in}, \phi_n^{out}$	Empirical parameters that capture the importance of a port as an exporter or importer.
κ	Number of sister vessels on the new service.
v	Vessel type to be deployed.
s	Speed of all vessels that will be deployed.

Problem $AUX(v, s, \kappa)$ has the following decision variables:

N_j	Binary variable, indicates whether port j is visited in the rotation.
B_j	Binary variable, indicates whether port j is a butterfly port in the rotation.
I_j	Continuous variable, used to indicate the sequence of port j in a rotation (for subtour elimination).
C_j	Binary variable, indicates if port j is the master port of the route.
A_{ij}	Binary variable, indicates whether edge (i, j) forms part of the new rotation.
Q_{od}	Continuous variable, indicates the number of FFEs with origin at port o and final destination at port d that will be carried per sailing of each vessel in the rotation.
W_1, W_2	Binary variables, respectively indicating whether the new rotation will have weekly or bi-weekly call frequency.
μ	<i>Inverse</i> of the number of trips to be completed over the entire planning horizon by each vessel on the new service.
ω	Estimated cost per sailing, per vessel, of the new service.

The auxiliary column generation model is then given by:

$$AUX(v, s, \kappa) \quad \text{maximize} \quad Z_{AUX(v, s, \kappa)} = s\kappa\omega \quad (2.11)$$

subject to

$$\omega = \mu(\tilde{f}^v - f^v) - \sum_{(i,j) \in \mathbf{E}} A_{ij} (eh^v p_j^v + eg^{vs} l_{ij}^v + d_j^v + a_{ij}^v) + \sum_{(o,d) \in \mathbf{G}} (q_{od} - u_o - u_d + \tilde{q}_{od}) Q_{od} \quad (2.12)$$

$$24T\mu = \sum_{(i,j) \in \mathbf{E}} A_{ij} \left(p_j^v + \frac{l_{ij}^v}{s} \right) \quad (2.13)$$

The objective function (2.11) maximizes the net revenue contribution from each sailing of the additional rotation. Constraint (2.12) defines the contribution from each sailing by prorating the daily running cost of the vessel, the opportunity/layup cost of the vessel, sailing and hotel fuel expenses, port calling fees, canal fees, and the revenues generated by the sailing. Constraint (2.13) establishes the number of trips to be completed by each vessel over the decision horizon as a function of sailing time between ports and port stay duration.

The following constraints establish the basic logic for variables A_{ij} , N_i , and B_i .

$$\sum_{j \in \mathbf{P}} A_{jn} = \sum_{j \in \mathbf{P}} A_{nj} \quad n \in \mathbf{P} \quad (2.14)$$

$$A_{ij} \leq (N_i + N_j)/2 \quad (i, j) \in \mathbf{E} \quad (2.15)$$

$$\sum_{j \in \mathbf{P}} B_j \leq 1 \quad (2.16)$$

$$\sum_{i \in \mathbf{P}} A_{ij} \leq N_j + B_j \quad j \in \mathbf{P} \quad (2.17)$$

$$\sum_{j \in \mathbf{P}} A_{ij} \leq N_i + B_i \quad i \in \mathbf{P} \quad (2.18)$$

Constraints (2.14) balance the number of arcs entering and leaving any port in the rotation. Constraints (2.15) allow an arc (i, j) to be part of the rotation only if ports i and j are part of the rotation. Constraints (2.16) limit to one the number of butterfly nodes in a rotation. Constraints (2.17) and (2.18) limit the number of arcs that can enter or leave a port. If a port is a butterfly port, two arcs may enter and leave the port. Otherwise, a single entry and departure are permitted for each sailing.

In order to establish an approximate balance between the cargo that will flow through the new rotation and the capacity of the vessels deployed to the rotation, we have the following constraints:

$$Q_{od} \leq \mu \hat{k}_{od} / \kappa \quad (o, d) \in \mathbf{G} \quad (2.19)$$

$$Q_{od} \leq \hat{k}_{od} N_o \quad (o, d) \in \mathbf{G} \quad (2.20)$$

$$Q_{od} \leq \hat{k}_{od} N_d \quad (o, d) \in \mathbf{G} \quad (2.21)$$

$$\sum_{d \in \mathbf{P}} Q_{od} \leq \phi_o^{out} c^v (N_o + \delta B_o) \quad o \in \mathbf{P} \quad (2.22)$$

$$\sum_{o \in \mathbf{P}} Q_{od} \leq \phi_d^{in} c^v (N_d + \delta B_d) \quad d \in \mathbf{P} \quad (2.23)$$

$$\sum_{(o,d) \in \mathbf{G}} l_{od}^v Q_{od} \leq \sum_{(i,j) \in \mathbf{E}} A_{ij} l_{ij}^v c^v \quad (2.24)$$

Constraints (2.19) limit the amount of cargo that can be carried between any o-d pair by each vessel on each sailing, by pro-rating the total remaining demand amongst all vessels sailing in the rotation, and the number of trips to be performed by each vessel. Constraints (2.20) and

(2.21) require ports o and d to be active in the rotation whenever any part of the residual demand for the corresponding o-d pair is carried by the candidate rotation. Our formulation does not represent the amount of cargo on board the vessels on every leg of the rotation. Rather, we use two approximations to balance the amount of cargo that is targeted for the proposed rotation against the capacity and number of vessels being deployed there. First, constraints (2.22) and (2.23) ensure that the amount of cargo loaded, respectively discharged, at any port d is less than a certain fraction ϕ_d^{out} , respectively ϕ_d^{in} , of the vessel capacity. The factors ϕ_d^{out} and ϕ_d^{in} capture the importance of each port as an importer or exporter of cargo relative to the overall residual demand for transport in the network. For the second approximation, we generate a lower bound (because we use the direct sailing distance for all o-d pairs) for the FFE-miles of cargo being transported. For the second approximation, we estimate the FFE-miles that are required by cargo being transported as well as the maximum FFE-miles that can be provided by the new rotation. The left-hand side of constraints (2.24) represents a (rather weak) approximation on the FFE-miles that will be consumed by the cargo being carried. The approximation is weak because we use the direct distance between origin and destination port l_{od}^v , whereas the cargo will more often travel indirectly, visiting several ports before reaching its destination. The right-hand side of constraints (2.24), however, provides an accurate measure of the FFE-miles that will be provided by the new rotation.

The subtour elimination constraints are:

$$\sum_{j \in \mathbf{P}} C_j = 1 \quad (2.25)$$

$$B_j \leq C_j \leq N_j \quad j \in \mathbf{P} \quad (2.26)$$

$$N_j \leq I_j \leq |\mathbf{P}|N_j \quad j \in \mathbf{P} \quad (2.27)$$

$$1 + I_i - |\mathbf{P}|C_j - |\mathbf{P}|(1 - A_{ij}) \leq I_j \quad (i, j) \in \mathbf{E} \quad (2.28)$$

Constraint (2.25) identifies exactly one port within the rotation as the base port. If the rotation has a butterfly node, constraint (2.26) requires that it must be at the rotation's base port. Constraints (2.27) forces variables I_j away from zero if and only if the corresponding port is to be visited by the rotation. Constraints (2.28) force the value of sequence number I_j to increase along the path of the rotation, except when returning back at the rotation's base port.

In order to ensure that all rotations have weekly or bi-weekly frequency we have the constraints:

$$W_1 + W_2 = 1 \quad (2.29)$$

$$W_2 = 0 \quad c^v \geq 1200\text{FFE} \quad (2.30)$$

$$(W_1 - 1) + \frac{5\kappa}{T} \leq \mu \leq \frac{7\kappa}{T} + (1 - W_1) \quad (2.31)$$

$$(W_2 - 1) + \frac{10\kappa}{T} \leq \mu \leq \frac{14\kappa}{T} + (1 - W_2) \quad (2.32)$$

Constraint (2.30) reflects highly competitive conditions in markets where larger vessels are deployed. This constraint requires that rotations employing vessels with capacity at or above 1200 FFE must have weekly calling frequency. It would be unwise to require rotation frequencies to be exactly seven or fourteen days. This would likely prevent the heuristic column generation process from obtaining many routes that are of commercial value. Additionally, many carriers build some schedule slack into their routes to address fluctuations in the weather or delays at one of the ports in the rotation. With this in mind, we write constraints (2.31) so that rotations with nominal weekly frequency might in fact call as often as every five days. Similarly constraints (2.32) permit that rotations with nominal bi-weekly frequency might call as often as every ten days.

Finally we have the integrality and non-negativity constraints:

$$A_{ij} \in \{0, 1\} \quad (i, j) \in \mathbf{E} \quad (2.33)$$

$$N_j, B_j, C_j \in \{0, 1\} \quad j \in \mathbf{P} \quad (2.34)$$

$$W_1, W_2 \in \{0, 1\} \quad (2.35)$$

$$Q_{od} \geq 0 \quad (o, d) \in \mathbf{G} \quad (2.36)$$

$$I_j \geq 0 \quad j \in \mathbf{P} \quad (2.37)$$

$$\mu, \omega \geq 0 \quad (2.38)$$

Our practical experience with problems of type $\text{AUX}(v, s, \kappa)$ is that these can be solved to optimality very quickly when the instance includes about 15 ports or less. Instances that entail more than 25 ports are significantly harder to solve, and are therefore not suitable for our overall strategy, where several thousand instances of $\text{AUX}(v, s, \kappa)$ may be launched. Our approach is to create clusters of ports that are tightly linked both geographically and by trade volumes. We sort such clusters according to the total demand for transport between the cluster's ports, and select the top cluster. We then formulate problems $\text{AUX}(v, s, \kappa)$ using up to 20 ports from the selected cluster. We believe that this approach results in a good compromise between the speed (allowing us to generate many rotations in the heuristic column generation) and solution quality (finding the best rotations).

We also note that the subtour elimination constraints (2.28) are an important source of the difficulty in solving problem $\text{AUX}(v, s, \kappa)$. In an effort to build up the network very quickly when the algorithm starts, we suppress constraints (2.28) for ten iterations. In many cases, we obtain solutions that do not contain subtours, allowing us to progress rapidly. As the network becomes more complex, however, it is necessary to reinstate constraints (2.28) in order to obtain valid solutions.

2.6.4 Complexity

In the following we will prove that the LSNDP is strongly NP hard by reduction from the Travelling Salesman Problem (TSP) in the general case and the set-covering problem for the case of the model (2.2)-(2.10). Agarwal and Ergun (2008a) proved their model to be weakly NP hard by reduction from the knapsack problem so the here presented proofs are stronger.

In the general case we may choose rotations arbitrarily. We show that LSNDP is strongly NP-hard by reduction from the TSP. The TSP may be defined as follows: Let $G = (N, A, C)$ be a graph where $N = \{1, \dots, n\}$ is the set of nodes, $A = \{(i, j) | i, j \in N, i \neq j\}$ is the set of edges and $C = c_{ij}$ is a cost or distance matrix associated to the edges A . An optimal solution to the TSP is a minimal cost Hamiltonian cycle covering the nodes in N .

Theorem 2.6.1. *The LSNDP in the general case is NP-hard*

Proof. We reduce from TSP. Let the set of ports P correspond to the set of nodes N in the TSP, and let the travel cost between ports correspond to the cost matrix C between nodes in the TSP. Moreover, set the demand between each pair of ports to 1, and limit the fleet list to one vessel of infinite capacity. The LSNDP will then choose the minimal cost Hamiltonian cycle between the ports P in order to satisfy all demands and is exactly an instance of the TSP. \square

If the rotations are given beforehand as in (2.2)-(2.10) it is easy to see that LSNDP is NP-hard by reduction from the set-covering problem. Given a number of nodes N and a family of sets S_1, \dots, S_ℓ , $S_i \subseteq N$ with corresponding costs $\bar{c}_1, \dots, \bar{c}_\ell$, the set covering problem asks to choose a subset of S_1, \dots, S_ℓ covering all nodes in N at the cheapest possible cost.

Theorem 2.6.2. *The LSNDP based on fixed rotations is NP-hard*

Proof. In order to reduce the set covering problem to LSNDP we let the set of ports be $\mathbf{P} = N \cup \{0\}$. For each subset S_i we introduce a rotation $R_i = S_i \cup \{0\}$ which visits the ports in arbitrary order.

The cost of a rotation is set to $\tilde{c}_i = \bar{c}_i$. There is one vessel type, with ℓ vessels each having capacity $|N|$. All transshipment costs are set to zero, and there is a demand of 1 between each port $i \in N$ and port 0. The revenue of delivering one unit is $q_{od} = 0$, while the penalty of not delivering one unit is $\tilde{q}_{od} = \infty$. The LSNDP will now choose the cheapest set of rotations covering all ports which is exactly the set covering problem. \square

2.7 Computational results

The model described in Section 2.6.1 has been solved heuristically for the benchmark data using the algorithm described in Section 2.6.2. The tests were performed on an Intel Xeon E5345, 2.66 GhZ Quad core with 20 GB RAM. As LP solver we have used Gurobi 4.5. The running time of the algorithm varies due to the large difference in the size of the instances. The maximal running times have been set experimentally. As described in Section 2.5.6 on page 38 there are three scenarios of each case representing high, medium and low capacity related to the base case referred to as **Low, Base, and High**. In Table 2.7, column **t sec.** reports the running time per case and reports algorithmic performance for the Median run of the **Low, Base, and High** cases. Ten replications of each instance have been made. The Median is given as the fifth worst value of the ten runs. All figures and remaining tables display the best and median solution with regards to the objective function. Please note that a profit will be negative, as we are minimizing the objective value, but are in reality maximizing revenue.

Several parameters in the heuristic are model and algorithm dependent. The planning period is set to 180 days and all demands are scaled accordingly. A weekly or biweekly frequency has been enforced in (2.31)-(2.32), and the first and second business rules of Section 2.4.2 are imposed. All bunker costs are fixed at 600 USD per metric Tonne. Port-draft incompatibilities are also considered in the model implementation.

Instance	t sec.	Columns evaluated			Unique columns			MCF Eval		
		Low	Base	High	Low	Base	High	Low	Base	High
Baltic	300	12912	12586	14046	1521	1719	1866	13680	13255	14809
WAF	900	13120	15343	12248	3858	4637	3951	13481	15343	12602
Mediterranean	1200	1582	2323	3192	1039	1197	1502	1634	2383	3244
Pacific	3600	4987	4203	4726	2552	2361	2660	5060	4268	4769
AsiaEurope	14400	4882	4291	4889	2322	2215	2806	4972	4357	4942
WorldSmall	10800	7313	7209	7628	2543	2937	3424	7425	7304	7731
WorldLarge	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table 2.7: Performance of the algorithm and the search progress for the median run with regards to the objective value. **Instance** denotes the instance, **t sec.** is the running time in seconds. **Columns evaluated** is the number of rotations evaluated in the tabu search, **Unique columns** is the number of unique rotations and finally **MCF eval** is the number of times the multicommodity flow solution was calculated. Note that we do not solve WorldLarge due to its size.

Tables 2.7 and 2.8 report the algorithmic performance, Tables 2.9 and 2.10 report the objective values and cost components and finally some network key performance indicators are reported in Tables 2.11 and 2.12.

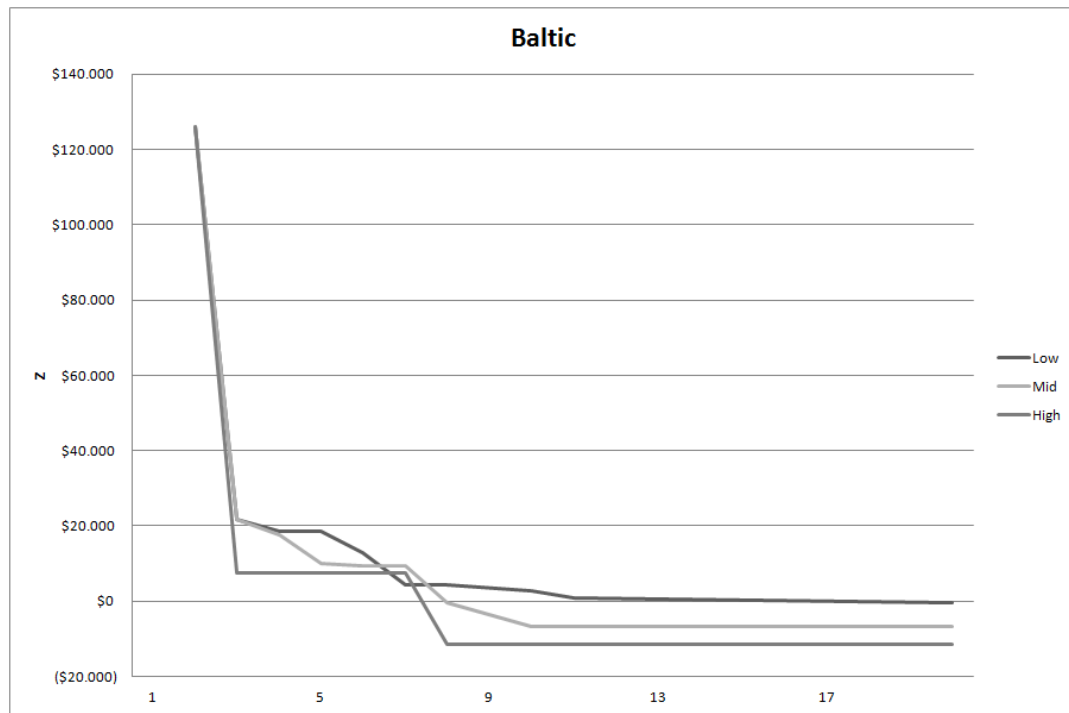
Figures 2.7(a) and 2.7(b) show the objective value for the median solution of the 10 randomized runs, as a function of the computational time, for the Baltic and AsiaEurope scenarios. For the remaining scenarios refer to Appendix A.

The figures show that the objective value is converging for most cases; rapidly for the smaller instances while for the larger instances there are still iterative improvements in spite of the increased run times. The ratio between unique columns(rotations) and total evaluated columns(rotations) is seen to increase with scenario size. This is due to the rotation generating MIP easily finding new unique columns in the larger search space of the larger instances, as opposed to the smaller

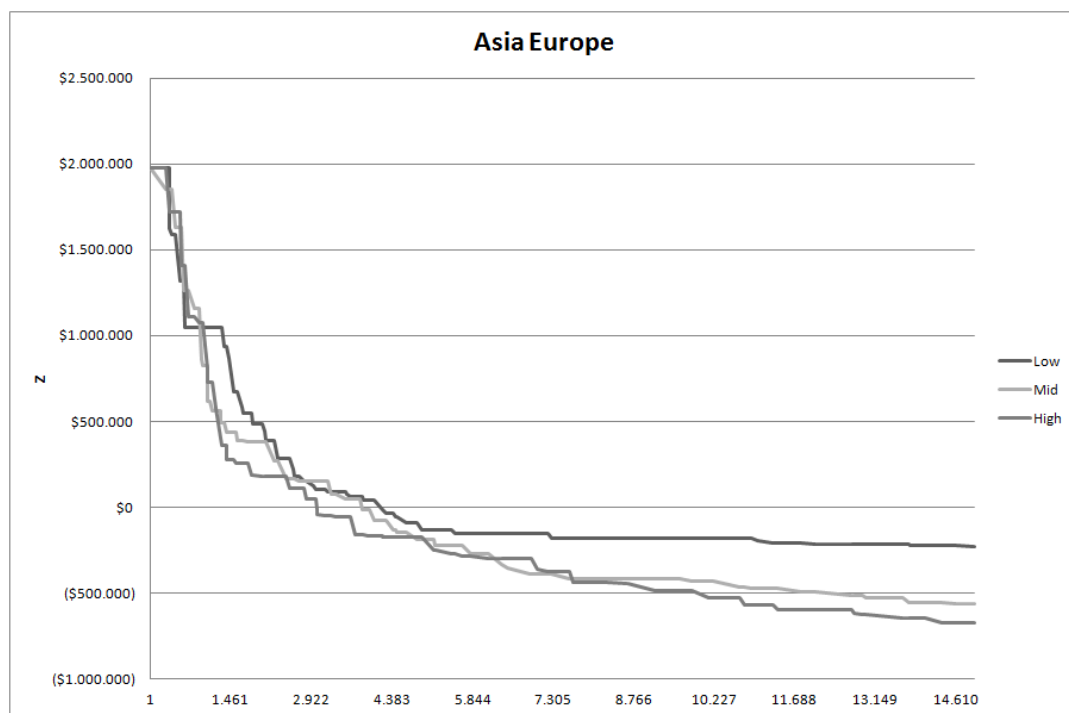
Instance	Iter			Improving Iter			Last Improving Iter		
	Low	Base	High	Low	Base	High	Low	Base	High
Baltic	548	358	459	12	12	9	151	30	12
WAF	267	337	271	19	30	26	115	195	221
Mediterranean	37	39	33	8	15	18	15	31	33
Pacific	57	54	33	31	26	30	57	30	33
AsiaEurope	102	80	87	64	60	58	102	80	85
WorldSmall	75	55	65	41	41	45	75	55	64
WorldLarge	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table 2.8: Performance of the algorithm and the search progress for the median run with regards to the objective value. **Instance** denotes the instance **Iter** is the number of iterations, where an iteration refers to \mathbf{R}^i , **Improving Iter** is the number of iterations, where an improving solution was found, and finally, **Last Improving Iter** is the last iteration, where an improving solution was found.

instances. The Mediterranean instance is an exception, where the ratio is rather high. It is believed to be due to a much denser demand matrix compared to the Baltic and WAF case. The number of iterations refers to \mathbf{R}^i of Section 2.6.2 and can be seen in Table 2.8. It can be seen that the number of iterations decreases with scenario size, even though computational time increases for the larger instances. That the solution space grows significantly with instance size can be seen, as we find more improving solutions even in the last iterations.



(a) Performance for Baltic case.



(b) Performance for AsiaEurope case

Figure 2.7: The objective value for the median Run Solution of the Low, Base and High instance of the Baltic and AsiaEurope case, as a function of the running time.

Instance			Z	Q	c_v	c_b	c_p	c_c	c_m	c_t	L_v	F
Baltic	Low	best	-6044	96750	7740	9522	8808	0	53780	0	0	10850
		median	838	92840	7740	9405	9509	0	51530	0	0	15490
	Base	best	-8365	98310	6480	10897	10440	0	55820	0	0	6306
		median	-6582	97860	6480	10208	10920	0	54760	62,49	0	8844
	High	best	-15678	103500	5760	11464	10520	0	59140	0	0	942,2
		median	-11509	101200	5760	9894	11670	0	57240	159,4	0	4954
WAF	Low	best	-115361	345800	57420	51750	15150	0	83130	3264	5940	25710
		median	-109470	336500	57420	43624	17270	0	79930	0	3960	32790
	Base	best	-143110	370600	52920	55447	20710	0	90760	3192	5760	10270
		median	-137681	355200	52920	44925	18940	0	84280	1690	8640	23360
	High	best	-159944	382000	48960	60519	16910	5827	94830	3859	10800	1956
		median	-152635	367800	48960	48756	19160	5827	88650	1376	11880	14330
Mediterranean	Low	best	29504	128900	27540	23649	18730	0	64030	10090	0	14360
		median	42636	121700	27540	27546	17320	0	62000	9694	0	20270
	Base	best	12209	136800	26640	22217	20190	0	69620	9475	1440	2311
		median	24934	134200	26640	23591	20610	0	68430	16630	1980	5187
	High	best	6606	137100	26100	22069	19330	0	69920	9861	5400	1807
		median	14267	136700	26100	25450	20260	0	70350	11720	4860	1904

Table 2.9: The objective value separated by cost and income components according to the model in section 2.6 on page 39. **Instance** denotes the name of the instance with a separate row for the **best** and **median** values over 10 replications. The **best** values correspond to the best solution obtained. Z is the objective value, note that we are minimizing expenses such that a negative value of Z is preferable, Q is the total revenue collected, c_v is the total vessel cost, c_b is the total fuel cost, c_p is the total port call cost, c_c are the canal fees, c_m is the total move cost at origin and destination nodes, c_t is the pure transshipment cost. L_v is the income from chartered out vessels and finally F is the sum of goodwill penalties for rejected cargoes. All costs are in k\$.

Instance			Z	Q	c_v	c_b	c_p	c_c	c_m	c_t	L_v	F
Pacific	Low	best	84615	1120000	276300	342536	31580	3180	429700	34610	2700	89080
		median	132020	1044000	276300	300382	32460	2989	390100	31450	0	142700
	Base	best	-54087	1197000	247700	348882	31610	6101	451600	34870	13500	35440
		median	-12774	1160000	247700	351830	35210	2967	444400	29110	17460	53710
	High	best	-101671	1208000	236500	350535	38340	6071	456400	33100	36720	22360
		median	-75321	1222000	236500	388324	36000	6016	460400	36180	31140	14150
WorldSmall	Low	best	-551842	4917000	1040000	1018924	100100	255400	1211000	199800	106200	646000
		median	-417100	4902000	1040000	1023956	103100	237900	1220000	247000	48060	661300
	Base	best	-1152761	5366000	942100	1114906	123800	271000	1336000	195200	160000	390500
		median	-888669	5199000	942100	1108834	126300	298400	1296000	213400	164000	489000
	High	best	-1291718	5550000	911700	1201543	129300	287800	1377000	263600	223200	310400
		median	-1168067	5266000	911700	1076555	109400	267100	1319000	244000	261500	431800
AsiaEurope	Low	best	-361041	3109000	718600	672023	117200	233600	609100	128800	17640	286900
		median	-228186	2998000	718600	661107	106800	228800	584400	125800	0	344000
	Base	best	-657972	3283000	648700	690406	134400	255700	637600	132700	81180	206700
		median	-561417	3179000	648700	726061	108800	254800	615600	132500	118100	248700
	High	best	-766385	3390000	625700	834862	130000	276700	656000	127900	176600	149400
		median	-670099	3181000	625700	708934	117700	253900	622400	131400	184900	235300

Table 2.10: The objective value separated by cost and income components according to the model in section 2.6 on page 39. **Instance** denotes the name of the instance with a separate row for the **best** and **median** values over 10 replications. The **best** values correspond to the best solution obtained. Z is the objective value, note that we are minimizing expenses such that a negative value of Z is preferable, Q is the total revenue collected, c_v is the total vessel cost, c_b is the total fuel cost, c_p is the total port call cost, c_c are the canal fees, c_m is the total move cost at origin and destination nodes, c_t is the pure transshipment cost. L_v is the income from chartered out vessels and finally F is the sum of goodwill penalties for rejected cargoes. All costs are in k\$.

Instance			dep%	R	PCpW	BPU%	WPU%	BAU%	WAU%	t/d%	rej%
Baltic	Low	Best	100,0	5	2,81	100,0	100,0	93,9	47,9	0,0	8,6
		Median	100,0	3	4,36	100,0	100,0	80,3	69,8	0,0	12,3
	Base	Best	100,0	4	4,06	100,0	100,0	75,9	43,2	0,0	5,0
		Median	100,0	5	2,96	100,0	100,0	92,9	48,2	0,3	7,0
	High	Best	100,0	5	3,48	100,0	100,0	98,6	56,6	0,0	0,7
		Median	100,0	4	4,12	100,0	100,0	83,6	61,5	2,7	3,9
WAF	Low	Best	89,4	7	5,19	100,0	95,1	90,4	50,4	7,0	11,7
		Median	92,9	8	4,04	100,0	100,0	79,4	49,7	13,5	14,9
	Base	Best	88,9	11	3,69	100,0	95,9	82,9	44,6	16,5	4,7
		Median	83,3	10	3,38	100,0	83,3	78,6	46,3	8,3	10,6
	High	Best	77,0	12	3,39	100,0	39,1	85,5	24,0	9,2	0,9
		Median	74,7	10	3,64	100,0	39,1	93,1	24,0	1,6	6,5
Mediterranean	Low	Best	100,0	7	5,97	100,0	100,0	94,4	70,1	56,8	7,4
		Median	100,0	7	5,57	100,0	100,0	95,3	76,9	50,4	10,4
	Base	Best	94,6	7	6,83	100,0	100,0	93,7	79,7	46,0	1,2
		Median	91,9	8	5,68	100,0	90,2	90,4	62,3	84,0	2,7
	High	Best	78,4	11	4,75	100,0	76,1	96,3	60,5	48,8	0,9
		Median	80,5	9	5,52	100,0	29,4	89,4	21,0	58,7	1,0

Table 2.11: Key Performance indicators for the smaller instances. **Instance** denotes the name of the instance with a separate row for the **best** and **median** values over 10 replications. The **best** values correspond to the best solution obtained. The **median** value is the median value of all KPI over the 10 replications. **dep%** is the percentage of the fleet deployed, **|R|** is the number of rotations in the final solution, **PCpW** is the average number of port calls per service per week, **BPU%**, **WPU%** is the best and worst peak utilization percentage respectively, **BAU%**, **WAU%** is the best and worst average utilization percentage respectively. **t/d%** is the percentage of transshipments performed of all delivered units and **rej%** is the percentage of rejected cargo.

Tables 2.11 and 2.12 give some key performance indicators for the networks generated. The *utilization percentage* is the relation between cargo transported and the available capacity on a vessel for a single voyage between two ports. The *Peak utilization* of a rotation relates to the voyage between two ports with the highest utilization percentage. *Best Peak Utilization (BPU)* denotes the peak utilization of the rotation with the highest peak utilization, and *Worst Peak Utilization (WPU)* the rotation with the lowest peak utilization. *The Average Utilization* relates to the average utilization percentage of all voyages on a rotation. *Best Average Utilization (BAU)* describes the average utilization of the rotation with the highest average utilization, and *Worst Average Utilization (WAU)* the average utilization of the rotation with the lowest average utilization.

Fleet utilization and rejected demand The percentage of the fleet deployed, **dep%**, seen in relation to the percentage of rejected demand, **rej%**, reveals lack of capacity to carry all demand for the low capacity instances as expected. Likewise, there is excess capacity in most high capacity instances with a corresponding low rejection rate. AsiaEurope and WorldSmall High instances are an exception to this pattern, which may be due to the profitability of cargo, but as the large instances are continuously finding improving rotations, solutions with a higher deployment and a decrease in rejected demand may exist.

Number of rotations The number of rotations are seen to increase with instance size. WorldSmall has a larger number of rotations than AsiaEurope correlated to the number of average port-calls on a service, **PCpW** being lower in WorldSmall than in AsiaEurope.

Utilization All solutions have a **BPU** of 100 % meaning that at least one service is fully utilized on one voyage between two ports. Low **WPU**s are also seen and can be acceptable for

Instance			dep%	R	PCpW	BPU%	WPU%	BAU%	WAU%	t/d%	rej%
Pacific	Low	Best	99,0	22	5,41	100,0	87,4	100,0	50,0	42,3	7,8
		Median	100,0	21	5,09	100,0	32,8	100,0	18,8	41,7	12,6
	Base	Best	95,0	21	5,73	100,0	36,6	100,0	21,0	46,0	3,1
		Median	93,3	21	5,54	100,0	58,8	100,0	38,5	44,8	4,7
	High	Best	85,1	20	6,53	100,0	100,0	100,0	50,0	47,5	2,0
		Median	87,0	24	5,02	100,0	7,7	100,0	5,3	39,0	1,2
WorldSmall	Low	Best	90,1	35	5,60	100,0	100,0	100,0	50,9	64,7	19,6
		Median	95,5	42	4,04	100,0	48,7	100,0	42,7	74,7	20,0
	Base	Best	83,0	41	4,87	100,0	37,0	100,0	28,3	64,7	11,8
		Median	82,8	40	4,67	100,0	22,1	100,0	21,1	65,3	14,8
	High	Best	76,5	41	4,68	100,0	24,0	100,0	23,1	73,7	9,4
		Median	71,9	37	5,06	100,0	79,6	100,0	52,7	78,3	13,1
AsiaEurope	Low	Best	97,5	30	6,39	100,0	34,0	98,5	22,4	80,2	14,5
		Median	100,0	32	5,50	100,0	32,5	100,0	25,7	75,3	17,4
	Base	Best	87,0	35	6,59	100,0	34,9	96,0	32,2	91,1	10,4
		Median	81,1	38	5,48	100,0	22,9	100,0	16,0	81,6	12,6
	High	Best	71,0	37	6,29	100,0	11,1	100,0	7,5	81,5	7,6
		Median	70,6	34	6,28	100,0	33,4	100,0	23,7	82,9	11,9

Table 2.12: Key Performance indicators for larger instances. **Instance** denotes the name of the instance with a separate row for the **best** and **median** values over 10 replications. The **best** values correspond to the best solution obtained. The **median** value is the median value of all KPI over the 10 replications. **dep%** is the percentage of the fleet deployed, **|R|** is the number of rotations in the final solution, **PCpW** is the average number of port calls per service per week, **BPU%**, **WPU%** is the best and worst peak utilization percentage respectively, **BAU%**, **WAU%** is the best and worst average utilization percentage respectively. **t/d%** is the percentage of transshipments performed of all delivered units and **rej%** is the percentage of rejected cargo.

feeder services for outlying profitable cargo. Looking at the average utilization, **BAU%**, **WAU%**, the networks are overall well utilized as some variation must be expected given the asymmetry of world trade and low utilization in some parts of the feeder network.

Transshipment percentage The percentage of number of transshipments over the demand units transported, **t/d%**, reveal that Baltic and WAF instances have very few transshipments and as expected, the Mediterranean case has a larger percentage of transshipments. For the Pacific, WorldSmall and AsiaEurope most demands are subject to one or more transshipments.

2.7.1 Discussion

Overall the solutions obtained are promising. The solutions utilize the available capacity well and the constructed networks transport the majority of the profitable cargoes, as can be seen in Tables 2.11 and 2.12.

Review by Network Planners Network planners at Maersk Line have evaluated that the given solutions are feasible liner shipping networks for the scenarios, but with room for improvement by inspection of experienced eyes. Some of these are dealt with by the incorporated business rules but other such as cabotage rules, commercially driven transit time restrictions, etc. is not dealt with. Realistic traits of the networks can be seen in the construction of shorter feeder services and longer inter continental services. Hubs are used realistically by connecting several feeders and inter continental services to the same hubs.

Baltic and WAF These small instances show very fast convergence in Figures 2.7 and A.1. The generated networks consist of services with few port calls (Table 2.11) (except lowly capacitated **Low WAF** instances), as expected of feeder networks with few or no transshipments as seen in Table 2.11.

Mediterranean Has a dense demand matrix which facilitates some hub structure and transshipments. No runs generated a profitable solution but the high capacity cases are close, the best with a loss of 6606 k\$. A study of the revenues of the demands reveal a rather low revenue compared to e.g. the baltic case and also the actual demand is spread on a large number of demand pairs meaning that the cargo compositions are more complex. The analysis indicates that the Mediterranean case is not very attractive from an economic standpoint.

Pacific and WorldSmall For these large instances complex networks are created with 20 to 40 services calling 5 to 6 ports on average. For Pacific most cargo is transported, except for the lowly capacitated instances, for the WorldSmall a little less is moved, but as vessels are chartered out it must be due to unprofitable demand, but still it is the most complex scenario and improvement are possible. The Pacific case has relatively few transshipments, due to the direct connections over the pacific and the high North American transshipment costs. WorldSmall has realistically, massive use of transshipments.

AsiaEurope The AsiaEurope case is a large challenge due to the size of the instance but also because the revenue is tight. From the utilization indicators (**BAU** / **WAU**) in Tables 2.11 and 2.12 it can be concluded that traffic flow is very intense in one direction displaying the trade imbalance, which is very decisive for network design today due to the excess capacity on the back-haul. This case is well suited for future work optimizing on empty repositioning. The amount of rejected cargo of the larger instances clearly indicate that some demand is in reality loss giving, a trait any real life case will have.

Cost Structure Looking at the median solution for the Base capacity largest scenario, WorldSmall, the network cost is distributed as 24 % for vessel costs, c_v , 28 % for fuel costs c_b , 10 % for port call and canal costs, $c_p + c_c$ and 38 % for move cost, $c_m + c_t$. Noting that this is a simplified case with demand aggregated to larger ports, reducing the needed number of transshipments and portcalls, this cost distribution can resemble the cost distribution of real liner shipping networks, although this of course fluctuates with cost and market changes.

Transshipments and Hubs The cases vary greatly in the hub-and-spoke structure and hence the desire to transship cargo. The Baltic case has very few transshipments as will be the case for a feeder network. In the AsiaEurope case all cargo transship more than once on average. Seen from a commercial point of view a transshipment carries a risk in terms of disruptions and an administrative overhead in transshipped cargo. These perceived “costs” of transshipping is not considered in the mathematical model and the found solution may have excess transshipments compared to real life. Adding customer satisfaction costs to the transshipment costs in the model can to some extent account for this problem.

Service Types The solutions clearly display the capacity levels of a feeder network and an inter-continental network, where larger vessels are servicing intercontinental routes. Network planners at Maersk Line confirm that the structure of the networks in terms of capacity deployment and the use of transshipment locations is fairly coherent with the operations of Maersk Line. Also the rotations vary greatly in design and display all of the design structures seen in Figure 2.1 and 2.2, showing the algorithms adaptability for different situations.

Fleet Size The different fleet size instances Low, Base and High capacity, results in different types of networks. The Low capacity instances obviously have less services, which on average are better utilized, as they will fill the vessels with the most profitable cargo and reject more cargo. The high capacity instances will have to create additional services to make a profit of the remaining cargo. Thus the resulting networks of the related fleet size scenarios will result in quite different networks and adds to the complexity of the benchmark suite.

2.8 Conclusion and future work

In this paper we have given an introduction to Liner Shipping Network Design for Operations researchers. The problem domain has been described and a discussion of mathematical modelling of the domain and the constraint set has been provided. A set of data instances was introduced as a benchmark suite, resembling real world problems. The problem has been proven to be strongly NP-hard and comparing with optimal methods for solving related Liner shipping Network Design models, it is apparent that this problem is among the most difficult network design problems. The model of Alvarez (2009) has been extended to handle the structure of a complex network, and a heuristic column generation algorithm has been presented using a new method for route generation. Computational results using the benchmark suite have been presented, and the solutions will be made available on a web-page in order to encourage a competition in developing the best algorithms. The algorithmic performance indicators suggest that we have created a benchmark suite that may be used for the development of both exact and heuristic approaches as the smaller cases should be suitable for benchmarking exact algorithms and the large instances poses a substantial challenge for state of the art heuristics. Overall, the benchmark suite provides a thorough test of a network design algorithm. The cases challenge the fleet deployment, port call sequence generation, the transshipment structure of the network and the selection of demand to transport in the network. At the same time the cases created closely resembles the real life operations of a global liner shipping company and the market for liner shipping in terms of trade imbalance, geographical transshipment points and fluctuations in demand compared to the capacity available. We also believe that the benchmark suite will prove valuable to future model development of the liner shipping network design problem. There is a large potential for future work on modelling as several commercial requirements such as low transit times and slow steaming have not been incorporated in the current network design models. The results have been evaluated by network designers of Maersk Line, who affirm that the traits of individual rotations resembles real world rotations, and that the overall network fulfill most of the relevant properties. It is also apparent by inspection, that the heuristic solutions may be improved upon, which encourages the development of better solution methods. We hope that this work will serve as a foundation for unifying and encouraging further work in Liner Shipping Network Design, which is a challenging real world problem having a huge impact on the cost of the world's supply chains. Moreover, the scientific world needs complex challenges in order to push algorithm development forward.

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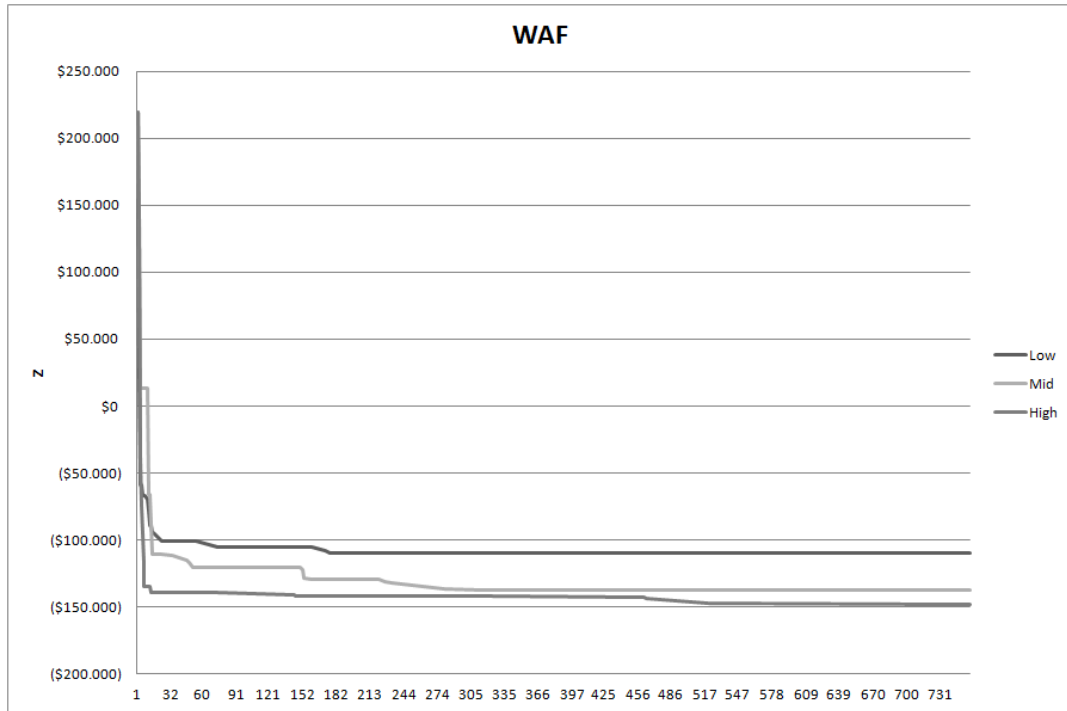
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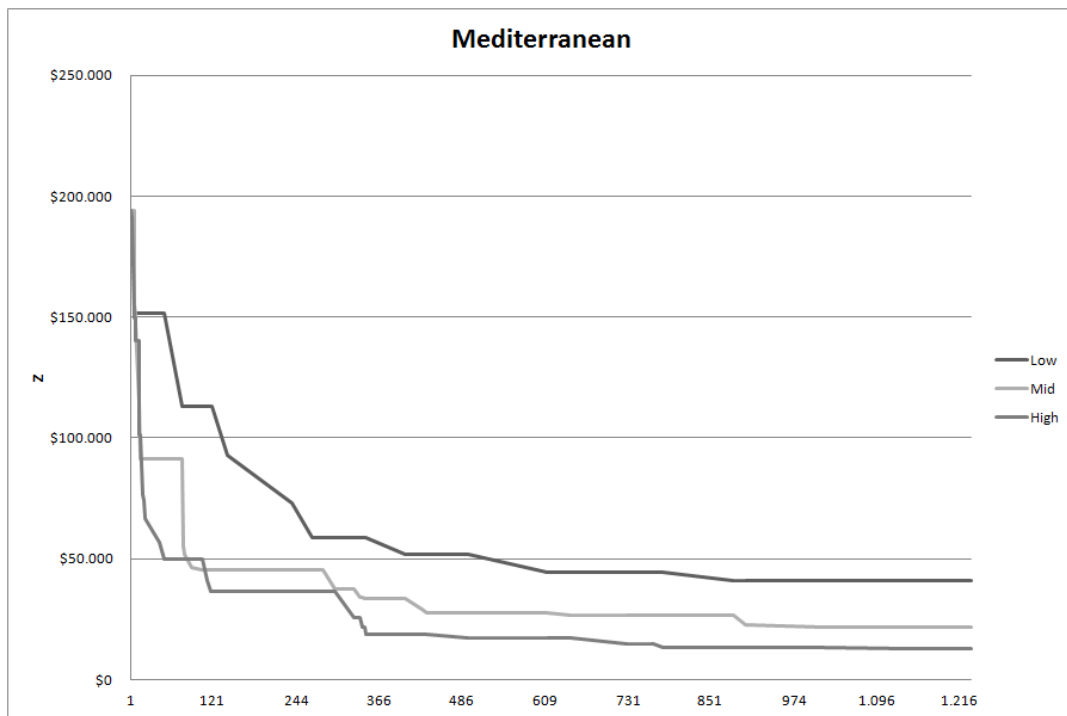
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Appendix A

Figures of remaining instances

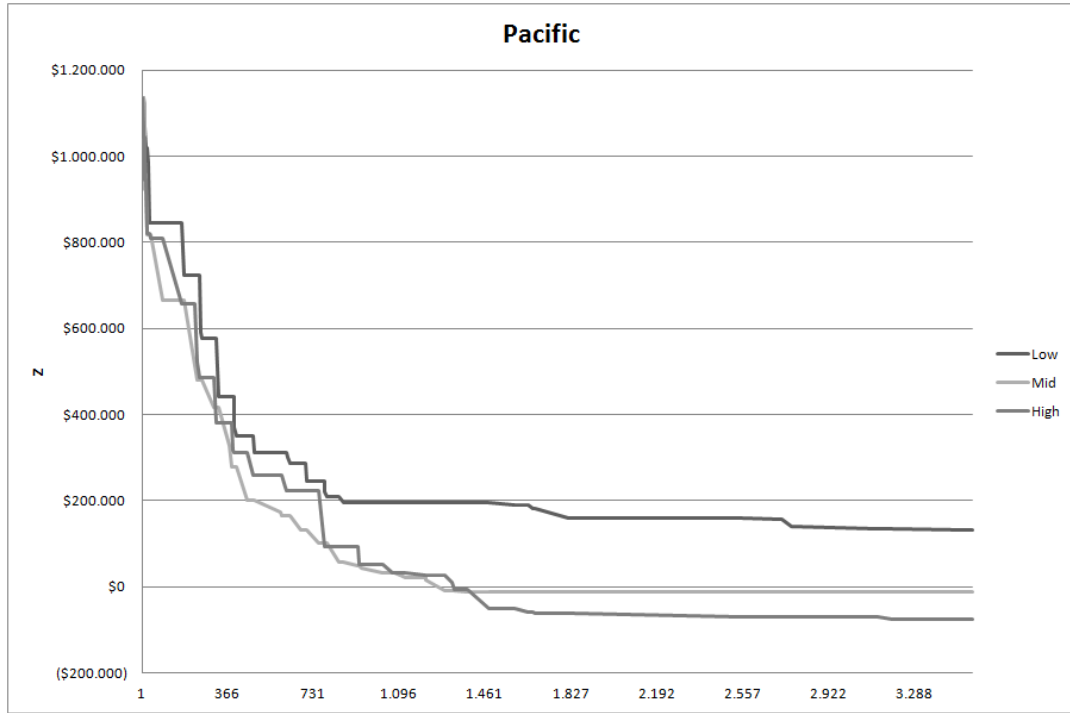


(a) Performance for WAF case

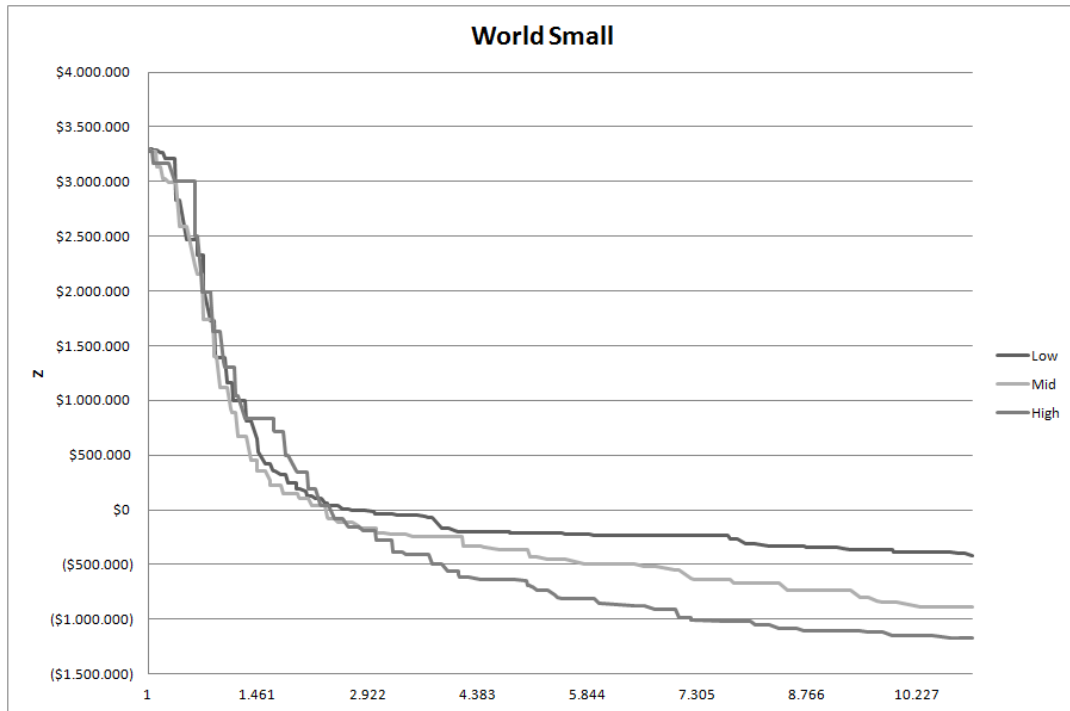


(b) Performance for Mediterranean case

Figure A.1: The objective value for the median Run Solution of the Low, Base and High instance, as function of the running time.



(a) Performance for Pacific case



(b) Performance for WorldSmall case

Figure A.2: The objective value for the median Run Solution of the Low, Base and High instance, as function of the running time.

Part II

Topics for optimization in liner shipping

Chapter 3

Liner Shipping Cargo Allocation with Repositioning of Empty Containers

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Abstract This paper is concerned with the cargo allocation problem considering empty repositioning of containers for a liner shipping company. The aim is to maximize the profit of transported cargo in a network, subject to the cost and availability of empty containers. The formulation is a multi-commodity flow problem with additional inter-balancing constraints to control repositioning of empty containers. In a study of the cost efficiency of the global container-shipping network, Song et al. (2005) estimate that empty repositioning cost constitutes 27% of the total world fleet running cost. An arc flow formulation is decomposed using the Dantzig-Wolfe principle to a path flow formulation. A linear relaxation is solved with a delayed column generation algorithm. A feasible integer solution is found by rounding the fractional solution and adjusting flow balance constraints with leased containers. Computational results are reported for seven instances based on real-life shipping networks. Solving the relaxed linear path flow model with a column generation algorithm outperforms solving the relaxed linear arc flow model with the *CPLEX* barrier solver even for very small instances. The proposed algorithm is able to solve instances with 234 ports, 16278 demands over 9 time periods in 34 minutes. The integer solutions found by rounding down are computed in less than 5 seconds and the gap is within 0.01% from the upper bound of the linear relaxation. The solved instances are quite large compared to those tested in the reviewed literature.

3.1 Introduction

This paper presents a cargo allocation model for strategic planning and network evaluation within a *liner shipping* company. A cargo allocation model is a strategic tool that, given a schedule and a fleet over time, evaluates network profitability concerned with routing profitable cargo in a fixed network incorporating the overall empty repositioning cost. The model is denoted the cargo allocation problem with empty repositioning (CAPER). CAPER is a multi-commodity flow problem similar to the revenue management booking model for air cargo transport presented by

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Bartodziej and Derigs (2004). However, air cargo is not container based as is the case for liner shipping. Therefore, we argue that a cargo allocation model for liner shipping must consider the cost of repositioning empty containers. Empty containers tend to accumulate at import intensive regions due to a significant imbalance in world trade. In a cost efficiency study of the global container trade, Song et al. (2005) estimate that empty repositioning cost accounts for 27% of the total world fleet running cost. Empty repositioning problems are well-studied in the literature on maritime transportation as a post optimization procedure given the allocation of cargo in the network. Several papers on pricing strategies in transportation markets with empty repositioning problems (Gorman, 2001; Topaloglu and Powell, 2007; Zhou and Lee, 2009) emphasize the problem of evaluating the empty repositioning problem separately. A network pricing perspective is supported experimentally in Gorman (2001): A 3.5 % increase in net profitability is possible by a 61% reduction in empty repositioning cost accepting a decrease in revenue and direct market profits: "To increase networkwide profits in this case, pricing managers must accept a reduction in both total revenues and direct market profits, which are more than recovered from the reduced repositioning cost" (Gorman, 2001). Literature within the area of liner shipping does not allow load rejection in general, although the limited capacity of a single liner shipper may require load rejection and the derived demand for empty repositioning may make it profitable. This paper presents the cargo allocation problem with load rejection and empty repositioning. The model enables exploration of several leasing policies. This allows for a network pricing perspective to strategically evaluate a given liner shipping network and select the most profitable set of cargo contracts given the derived empty repositioning and equipment leasing cost.

The mathematical model of CAPER is an augmented multi-commodity flow problem where inter-balancing constraints (Crainic et al., 1989) ensure repositioning or leasing of empty containers at ports with a positive net flow. In a multi-commodity flow problem a graph $G = (N, A)$ and a set of commodities K is given. The classical formulation of the standard multi-commodity flow problem is the arc flow formulation with $|K||A|$ variables and $|A| + |K||N|$ constraints due to flow conservation at every node. Although the number of variables is polynomially bounded, representing a global shipping network is too large for standard linear programming (LP) solvers. Dantzig-Wolfe decomposition can be applied to generate a path flow formulation with only $|A| + |K|$ constraints. However, the number of variables in the path flow formulation may be exponential. To circumvent this problem we use delayed column generation, as it can be proven that at most $|K| + |A|$ paths carry positive flow (Ahuja et al., 1993). The pricing problem is a shortest path problem, where the cost of a path represents the reduced cost of a path variable. Due to the structure of the dual problem, the arc costs of the pricing problem for the path flow model are positive, which results in a polynomially solvable pricing problem when considering column generation. Ahuja et al. (1993) argue that delayed column generation on multi-commodity flow problems converge slowly and that an interior point method on the arc flow formulation is superior for large scale LPs. Experimental results performed in this study indicate that column generation outperforms interior point methods even for small instances of CAPER. This is believed to be due to the structure of the liner shipping network design, which results in a sparse graph with few competitive routings for each commodity.

As containers cannot be split, CAPER is an integer multi-commodity flow problem which is \mathcal{NP} -hard. Because the number of containers transported in a global shipping network is huge, rounding down the fractional part of demands may be considered insignificant. This is confirmed by experimental results in this paper. A nice property of the path flow formulation is that flow conservation constraints are implicitly satisfied on a path. Hence, a feasible integer solution can be obtained by rounding down all fractional variables and supplying empty containers through a leasing variable at nodes with violated inter-balancing constraints. Therefore, we consider the linear relaxation of the path model of CAPER and obtain a heuristic integer solution by rounding down the LP solution of the path flow model. The contribution of this paper is threefold:

- To present an augmented multi-commodity arc flow formulation of the cargo allocation problem with load rejection and empty repositioning. The arc flow formulation is decomposed into an equivalent path flow formulation using the Dantzig-Wolfe decomposition principle.

<i>Publication</i>	<i>Routes</i>	<i>Method</i>	<i>Specialities</i>	<i>Ports</i>
Reinhardt and Kallehauge (2007)	multiple	branch-&-cut	transshipment	20
Agarwal and Ergun (2008)	multiple	greedy, column generation, benders	transshipment	10
Alvarez (2009)	multiple	tabu search	transshipment	120
Shintani et al. (2007)	single	genetic algorithm	empty repositioning, bunker consumption	20

Table 3.1: Recent literature within the field of liner shipping network design. *Routes* indicate whether single or multiple services are optimized. *Method* indicates the algorithm applied. *Specialities* concern additional cost and constraints considered. *Ports* indicate the number of ports in the test instances solved.

- To demonstrate experimentally that our column generation algorithm is computationally tractable and attractive for solving large scale instances of CAPER. The instances generated are derived from real-life liner service networks. The computational results show that delayed column generation scales well for large instances. Instances with up to 234 ports and 16273 demands over 9 time periods were solved in less than 34 minutes with the column generation algorithm. The largest instance solved for 12 time periods contains 22433 commodities between 222 ports and was solved in approximately 71 minutes. It is shown that high quality integer solutions within 0.01% from the LP upper bound of the path flow formulation can be found by a simple rounding heuristic.
- Lastly, the computational results indicate that delayed column generation outperforms interior point methods for the cargo allocation problem. The cargo allocation problem is believed to have a special structure as the graph is sparse. The structure may aid the convergence of delayed column generation although this cannot be concluded.

This paper is organized as follows: The following section describes related work. Section 3.3 describes the network representation used throughout this paper. Section 3.4 presents the arc flow model, and Section 3.5 presents the decomposed path flow model and the pricing problem used in the delayed column generation algorithm. Section 3.6 reports our computational results on seven generated test instances. Section 3.7 provides some concluding remarks and future work on CAPER.

3.2 Literature Overview

Excellent reviews on operations research in shipping up until 2007 is found in (Christiansen et al., 2004, 2007). Literature on optimization problems within liner shipping is particularly scarce, but interest in the research area is growing. The area of liner shipping network design (Reinhardt and Kallehauge, 2007; Agarwal and Ergun, 2008; Alvarez, 2009; Imai et al., 2009) has received increasing attention in recent years. The cargo allocation problem, which is the subject of this paper, is considered part of the liner shipping network design problems studied by (Reinhardt and Kallehauge, 2007; Agarwal and Ergun, 2008; Alvarez, 2009). The much harder problem of designing the network depends on the cargo allocation problem. All papers apply a multi-commodity arc-flow formulation of the cargo allocation problem without consideration for empty repositioning. The paper of Alvarez (2009) is to the best of our knowledge the only study of large scale instances of the liner service network design problem. In Alvarez (2009) the cargo allocation problem is solved as a subroutine of the tabu search algorithm solving the network design problem. Alvarez calls for faster algorithms for solving the cargo allocation problem.

Shintani et al. (2007) presented a model for routing a single service in Asia considering empty repositioning. Load rejection is not allowed at ports called and hence empty repositioning is conditioned by excess capacity on vessels. Repositioning cost is the penalty cost of storing or leasing empty containers. Shintani et al. (2007) conclude that empty repositioning is decisive for the calling sequence and bunker consumption. An overview of the literature within liner shipping network design and the size of test instances for experimental problems is presented in Table 3.1.

Publication	Focus	Environment	problem sizes
Erera et al. (2009)	Robust optimization	Empty repositioning problems	20 ports, 600 tank containers
Francesco et al. (2009)	Multi-scenario policies	Empty repositioning problems	Regional size network, no numbers
Dong and Song (2009)	Joint optimization	Container fleet size considering empty repositioning	3 ports, 1 voyage, 550 TEU
Feng and Chang (2008)	Regional repositioning plans	Port-to-port empty repositioning	33 ports, 17 voyages, 3380 TEU, 880 FFE, 1146 FFE HQ
Lam et al. (2007)	Approximate dynamic programming	Empty repositioning problems	3 ports, 9 voyages
Jula et al. (2006)	Empty repositioning strategies	Empty container reuse at LA/LB port	Computational results not comparable
Li et al. (2007)	Reduce holding of empty containers	Single port inventory model	1 port, 100 TEU
Li et al. (2004)	Reduce holding of empty containers	Multiple port inventory model	3 ports, 226 TEU
Shen and Khoong (1995)	Repositioning and leasing strategy	Decision support system at port level	NC
Cheung and Chen (1998)	Two stage stochastic network model	Empty repositioning problem	10 ports, 6 voyages
Crainic et al. (1993)	Empty repositioning strategy	Empty repositioning problem	NC

Table 3.2: Recent literature within the field of empty repositioning. FFE= Forty-foot Equivalent Unit (The standard measurement unit of containerized cargo), TEU=Twenty-foot Equivalent Unit, HQ= High Cube container, NC= No computational results.

Empty container repositioning for maritime transport has received significant attention (Crainic et al., 1993; Shen and Khoong, 1995; Li et al., 2004, 2007; Shintani et al., 2007; Erera et al., 2005, 2009; Francesco et al., 2009; Imai et al., 2009). Erera et al. (2005) study the empty repositioning of a fleet of tank containers in a time-extended uncapacitated network. They advocate the simultaneous optimization of routing current bookings and the derived empty repositioning to achieve higher utilization of tank containers and minimize cost for the operator. Computational results are presented to prove that the simultaneous optimization of routing bookings and empty repositioning is possible with standard solvers for a problem with 900 commodities over a six month planning horizon.

(Crainic et al., 1993; Shen and Khoong, 1995; Li et al., 2004, 2007; Shintani et al., 2007; Imai et al., 2009; Francesco et al., 2009; Erera et al., 2009) are concerned with empty repositioning plans based on known or forecasted demand and supply at ports. An overview of the literature on empty repositioning along with the size of experimental results is provided in Table 3.2.

Francesco et al. (2009) present an operational empty container repositioning model. The model minimizes the repositioning cost as the total sum of transportation, storage, loading and unloading cost. There is specific focus on storage capacities and costs as these may vary between ports. The model is used to investigate multi-scenario policies for repositioning given the uncertainty of historical data and demand. Li et al. (2004) present the empty container problem as a non standard inventory model for a single port to reduce holding of redundant empty containers. The model is extended to multiple ports with a repositioning plan in Li et al. (2007).

Lam et al. (2007) present a dynamic container allocation model based on approximate dynamic programming. Jula et al. (2006) studies empty container reuse on the land-side of the Los Angeles/Long Beach ports. The aim is to reduce congestion at the port at a lower overall inland repositioning cost and reduce pollution in the Los Angeles area through alternative repositioning strategies. Erera et al. (2009) present a robust optimization framework for dynamic empty repositioning problems acknowledging the uncertainty of demand. Feng and Chang (2008) present a study of empty repositioning along with a routing plan for the repositioning of empty equipment. The study is concerned with intra-Asia transport. The aim is to free long haul vessel slots for loaded traffic. Dong and Song (2009) present a model for the joint optimization of container fleet size and empty repositioning. Song et al. (2005) studies the global container fleet in order to study the impact of empty repositioning. The study concludes that empty repositioning cost constitutes 27% of the total world fleet running cost. Crainic et al. (1993) present a dynamic and stochastic

model for the empty container allocation problem. The context is an international shipping company with focus on the land operations, i.e., movements between customers and depots. Shen and Khoong (1995) present a decision support system for empty container distribution planning for a shipping company at port level. Cheung and Chen (1998) have developed a two stage stochastic network model for the dynamic empty container allocation problem. The model is highly related to Crainic et al. (1993).

Crainic et al. (1989) present a multi mode multi-commodity location-distribution problem with inter-depot balancing requirements. The model is primarily a location problem deciding the number and locations of inland depots for empty vehicles. However, it also determines the empty flows between depots according to the inter-balancing constraints.

The network pricing perspective is explored in Gorman (2001) by a case study of the Burlington Northern and Santa Fee Railway (BNSF). An economic pricing model considering derived empty repositioning cost implied a rationale in decreasing direct market profit to achieve significant empty repositioning cost, which could cater for the reduction in direct market profit. The result for BNSF was an increase of 5% in loaded miles and a 3% reduction in repositioning from 1998 to 2001. Topaloglu and Powell (2007) study the coordination of pricing decisions and fleet management in a freight transportation market. They emphasize that profit maximization of separate origin-destination must incorporate the network downstream effect of empty repositioning. Zhou and Lee (2009) study pricing and competition in a transportation market with empty repositioning.

As can be seen from Table 3.2 repositioning of empty containers has been studied from different perspectives and has been approached by several solution methods. Only Erera et al. (2005) handle loaded and empty containers simultaneously. Although CAPER is related to Erera et al. (2005) our model differs in several respects. First, Erera et al. (2005) does not allow load rejection and only seek to minimize the cost of transport and empty repositioning. Second, tank containers must be cleaned at a designated site before reuse. Third, Erera et al. (2005) transport the tank containers in an uncapacitated intermodal network as opposed to the capacitated liner shipping network of this paper. The liner shipping company owns the fleet of containers and wish to reposition them using the capacity of its own network. Lastly, our algorithm solves instances with up to 16000 commodities over a twelve month planning period within one hour. However, the problem of Erera et al. (2005) is solved to integer optimality using standard solvers as opposed to the rounded integer solution presented here, but this is necessary due to the large scale. The results of Erera et al. (2005) confirm the economic rationale in simultaneously considering loaded and empty containers.

The papers on empty repositioning in Table 3.2 are mainly focused on operational repositioning plans. CAPER is considered a strategic tool for network evaluation and cargo selection, where empty container repositioning is seen as a significant cost component of the cargo allocation problem. The transport of empty and loaded containers share the capacity of the network, and demand for empty containers is derived from the allocation of loaded containers. Crainic et al. (1993) wish to jointly optimize loaded and empty container allocation, but reject the idea due to the computational complexity of the problem. Erera et al. (2005) argue that the joint optimization of loaded and empty container allocation is feasible today due to the advances in both mathematical programming as well as computer hardware and software. This is confirmed by the computational results of this paper and that of Erera et al. (2005). The experimental results of this paper demonstrate that it is computationally tractable to simultaneously optimize loaded and empty movements even for large scale instances. The size of the instances solved in this paper is significantly larger than the instances presented in the literature of liner shipping network design problem (see Table 3.1) as well as for empty repositioning problems (see Table 3.2).

3.3 Network Representation

CAPER operates on a capacitated network representing the service network of a liner shipping company. The liner shipping network consists of a set of unique ports P connected by the services S offered by the liner shipping company. All services are cyclic and may take several months

to rotate. The network is modeled over time to allow for varying planning periods and a rolling horizon planning scheme. In the CAPER network the time periods are divided into time slots of length equal to the greatest common divisor for all voyage times between two ports, e.g., one day. Obviously, a detailed network can become very large and may therefore be intractable for practical purposes. Hence we propose to aggregate the time periods into larger time slots to produce a smaller network. For strategic planning this is assumed to be reasonable. Aggregation of the network is discussed at the end of this section.

A *time-space* network is created as a capacitated directed acyclic graph $G = (N, A)$. Let T be the set of time periods and let the node set be $N = \{p^t \mid p \in P, t \in T\}$. The arc set $A = A_G \cup A_R$ consist of a set of uncapacitated *ground* arcs $A_G = \{(p^t, p^{t+1}) \mid p^t, p^{t+1} \in N, t \in T\}$ representing the stock at a port p between two subsequent time periods, and a set of capacitated *travel* arcs $A_R = \{(p^t, q^{t+\tau_{pq}}) \mid (p^t, q^{t+\tau_{pq}}) \in N, t \in T, p \neq q, p, q \in P, u(p^t, q^{t+\tau_{pq}}) > 0\}$ representing a travel between two different ports p and q with travel time τ_{pq} such that the capacity is more than 0. Let S be the set of services such that C^s is the capacity of the vessel belonging to service s and let $A^s = \{(p^0, q^{0+\tau_{pq}}), \dots, (r^t, p^{t+\tau_{rp}}) \mid p, q, r \in P\}$ be the set of arcs representing the sequence of ports in the rotation starting at the first time period and ending at time T^S . Assume that all rotations start in the first time period. The capacity of an arc $(p^t, q^{t+\tau_{pq}}) \in A_R$ is

$$u(p^t, q^{t+\tau_{pq}}) = \sum_{s \in S} \sum_{(p^i, q^j) \in A^s} C^s$$

where $i = t \bmod T^S$ and $j = (t + \tau_{pq}) \bmod T^S$. That is, the capacity $u_{i,j}$ of arc $(i, j) \in A_R$ is the accumulated vessel capacity of all services with a voyage on arc (i, j) .

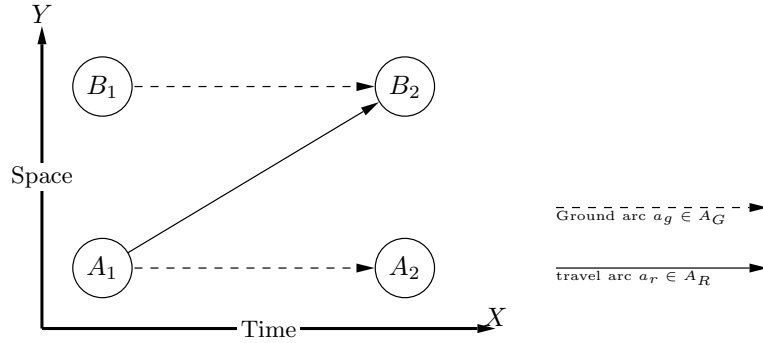


Figure 3.1: Example of time-space network.

Consider aggregating time periods T into larger time slots each with h of the smaller time slots creating a set of new time periods T' . A time slot $t \in T$ is aggregated into the new time slot $t' \in T'$ by integer division with h , that is t goes into the aggregated time slot $t' = \lfloor t/h \rfloor$. This results in an aggregated graph with a factor h less nodes $G' = (N', A')$ where the node set is $N' = \{p^{t'} \mid p \in P, t' \in T'\}$, and the arc set is $A' = A'_G \cup A'_R$. The aggregated ground arcs are $A'_G = \{(p^{t'}, p^{t'+1}) \mid p^{t'}, p^{t'+1} \in N', t' \in T'\}$ and the aggregated travel arcs

$$A'_R = \left\{ (p^{i'}, q^{j'}) \left| \begin{array}{l} p^{i'}, q^{j'} \in N', \\ p \neq q, p, q \in P, \\ i', j' \in T', \\ \exists (p^i, q^j) \in A_R, \text{ where } i' = \lfloor i/h \rfloor, j' = \lfloor j/h \rfloor, i, j \in T \end{array} \right. \right\}.$$

The capacity of the aggregated travel arc $(p^{i'}, q^{j'})$ is

$$u(p^{i'}, q^{j'}) = \sum_{(p^i, q^j) \in A_R} u(p^i, q^j)$$

where $i' = \lfloor i/h \rfloor$ and $j' = \lfloor j/h \rfloor$. That is, the capacity of the aggregated arc is a sum of the capacities of the original arcs that have been aggregated. Note that the aggregated graph is not

necessarily acyclic. However, due to the objective function the commodities do not cycle since it will result in an increased expense.

3.4 Arc Flow Formulation

A set of commodities K is transported in the liner shipping network represented by the time-space graph described in section 3.3. A commodity is defined as the tuple (O_k, D_k, d_k, r_k) representing a demand of d_k in number of containers from node $O_k = p^t$ to node $D_k = q^{t'}$ with a sales price per unit of r_k . The unit cost of arc (i, j) for commodity k is denoted c_{ij}^k . The non-negative integer variable x_{ij}^k is the flow of commodity k on arc (i, j) . The capacity of arc (i, j) is u_{ij} .

The standard multi-commodity flow model does not consider the supply of empty containers. Inter-balancing constraints are applied to every node to account for availability of empty containers. The constraints require that every node is balanced with regards to the in- and out-flow of containers. Hence, the current allocation of loaded commodities results in a derived demand for empty containers. The empty containers may flow between any nodes and the derived demand from the current cargo allocation decides origin, destination and quantity of empty containers. To model the empty containers an empty super commodity k_e is introduced. The flow of the empty super commodity is defined $\forall (i, j) \in A$ as the integer variables $x_{ij}^{k_e}$. The unit cost of arc (i, j) for commodity k_e is denoted $c_{ij}^{k_e}$. The empty super commodity has no flow conservation constraints and appear in the objective with a cost and in the bundled capacity and inter-balancing constraints. For convenience the commodity set is split into the loaded commodities and the empty super commodity: Let K_F be the set of loaded commodities. Let K_e be the set of the single empty super commodity. Finally, let $K = K_F \cup K_e$. The inter-balancing constraints also introduce a new set of variables representing leased containers at a node. The cost of leasing is modeled in the objective. Let c_l^i be the cost of leasing a container at port i , while l_i is the integer leasing variable at port i . Demand may be rejected, due to capacity constraints and unprofitability from empty repositioning cost. The slack variable γ_k represents the amount of rejected demand for commodity k . The cost of transport, leasing and derived demand for the empty super commodity must be subtracted. The arc flow model of CAPER with a profit maximizing objective, an empty super commodity, leasing variables, load rejection and inter-balancing constraints is stated as:

$$\max \sum_{k \in K_F} \sum_{j \in N} r_k (d_k - \gamma_k) - \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k - \sum_{i \in N} c_l^i l_i \quad (3.1)$$

$$\text{s.t.} \quad \sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k + \gamma_k = d_k \quad i = O_k \quad k \in K_F \quad (3.2)$$

$$\sum_{j \in N} x_{ji}^k - \sum_{j \in N} x_{ij}^k + \gamma_k = d_k \quad i = D_k \quad k \in K_F \quad (3.3)$$

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k = 0 \quad i \in N \setminus \{O_k, D_k\} \quad k \in K_F \quad (3.4)$$

$$\sum_{k \in K} x_{ij}^k \leq u_{ij} \quad (i, j) \in A \quad (3.5)$$

$$\sum_{k \in K} \sum_{j \in N} x_{ij}^k - \sum_{k \in K} \sum_{j \in N} x_{ji}^k - l_i \leq 0 \quad i \in N \quad (3.6)$$

$$x_{ij}^k \in \mathbb{Z}_+ \quad k \in K, (i, j) \in A \quad (3.7)$$

$$l_i \in \mathbb{Z}_+ \quad i \in N \quad (3.8)$$

$$\gamma_k \in \mathbb{Z}_+ \quad k \in K_F \quad (3.9)$$

The objective (3.1) is to maximize the profit of the demanded flow of all commodities in K on the arcs. Constraints (3.2)-(3.4) are the flow conservation constraints which guide the flow of a

demand from origin to destination. Furthermore, the flow and the rejected demand must equal the demanded quantity d_k . Constraints (3.5) are the bundle constraints ensuring that the flow of all commodities do not exceed the capacity of the arcs. Constraints (3.6) are the inter-balancing constraints which give a derived demand for the empty super commodity and leased containers. It is possible to raise the bound on (3.6) for nodes in the first time period ($t = 1$) to reflect the current stock of empty equipment. Constraints (3.7)-(3.9) ensure non-negative and integral flow and leasing variables. The formulation is polynomial in the input size as the number of variables is $O(|K||A| + |N|)$ and the number of constraints is $O(|N||K| + |A| + |N|)$.

It should be noted that when a container is leased, it remains in the network for the rest of the period. This corresponds to long-term leasing for the first period and short term leasing in the last periods. The cost of a leasing variable should depend on the amount of time periods remaining in T at the node where it is leased. Off-leasing can be modeled by an off-leasing variable making cost dependent on the net leasing between in- and off-leasing variables at the nodes. This changes the objective function (3.1) to:

$$\max \sum_{k \in K_F} \sum_{j \in N} r_k(d_k - \gamma_k) - \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k - \sum_{i \in N} c_l^i (l_{in}^i - l_{off}^i)$$

and the inter-balancing constraints (3.6):

$$\sum_{k \in K} \sum_{j \in N} x_{ij}^k - \sum_{k \in K} \sum_{j \in N} x_{ji}^k - l_{in}^i + l_{off}^i \leq 0 \quad i \in N$$

The above model assumes off-leasing can occur at any port. Off-leasing can be restricted to certain ports by defining off-leasing variables accordingly. When the container is leased, the remainder of the optimization period is paid for. When it is off leased, the remainder of the optimization period at the off-leasing point is refunded.

Various services are offered by leasing companies that own half the maritime container fleet worldwide, see (IICL). Leasing services vary from one-trip and round-trip leases to short-, medium- and long-term leasing ranging from one month to 42 months, see (TAL).

The experimental tests in section 3.6 apply the original model allowing only in-leasing at nodes. As a result leasing may range from a single time period to the entire time period optimized upon. The model allows exploration of different leasing strategies by altering variables for in- and off-leasing.

3.5 Path Flow Formulation

In the following we introduce a path flow model for CAPER. The arc flow model of CAPER (3.1)-(3.9) has block-angular structure with $|K_F|$ subproblems given by the flow conservation constraints for each full commodity. The commodity subproblems are tied together by the bundle constraints (3.5), i.e., the arc capacity constraints, and the inter-balancing constraints (3.6) regarding the supply of empty containers. The structure of the arc-flow model enables us to present an equivalent path flow model by applying the Dantzig-Wolfe decomposition principle (Dantzig and Wolfe, 1961).

3.5.1 Path flow reformulation

Using Dantzig-Wolfe decomposition we get a master problem considering paths for all commodities, and a subproblem defining the possible paths for each commodity $k \in K$. Due to the *flow decomposition theorem* (Chapter 3.5 in (Ahuja et al., 1993)), which states that every non-negative arc flow can be represented as a non-negative path and cycle flow, this can be formulated such that master problem variables define the flow of a commodity on a path, and the subproblem finds valid paths for that commodity.

Let p be a path connecting O_k and D_k and P_k be the set of all paths belonging to commodity k . The flow on path p is denoted by the variable $f(p)$. The binary indicator $\delta_{ij}(p)$ is one if and only

if arc (i, j) is on the path p . Finally, $c_p^k = \sum_{(i,j) \in A} \delta_{ij}(p) c_{ij}^k$ is the cost of path p for commodity k . The master problem is:

$$\max \sum_{k \in K_F} \sum_{p \in P_k} (r_k - c_p^k) f(p) - \sum_{(i,j) \in A} c_{ij}^{k_e} x_{ij}^{k_e} - \sum_{i \in N} c_l^i l^i \quad (3.10)$$

$$\text{subject to } \sum_{k \in K_F} \sum_{p \in P_k} \delta_{ij}(p) f(p) + x_{ij}^{k_e} \leq u_{ij} \quad (i, j) \in A \quad (3.11)$$

$$\sum_{p \in P_k} f(p) + \gamma_k = d_k \quad k \in K_F \quad (3.12)$$

$$\sum_{k \in K_F} \sum_{p \in P_k} \sum_{j \in N} (\delta_{ij}(p) - \delta_{ji}(p)) f(p) + x_{ij}^{k_e} - x_{ji}^{k_e} - l^i \leq 0 \quad i \in N \quad (3.13)$$

$$f(p) \in \mathbb{Z}_+ \quad p \in P_k \quad k \in K_F \quad (3.14)$$

$$\gamma_k \in \mathbb{Z}_+ \quad k \in K_F \quad (3.15)$$

$$x_{ij}^{k_e} \in \mathbb{Z}_+ \quad (i, j) \in A \quad (3.16)$$

$$l^i \in \mathbb{Z}_+ \quad i \in N \quad (3.17)$$

Where the x_{ij}^k variables are replaced by $\sum_{p \in P_k} \delta_{ij}(p) f(p)$ according to the flow decomposition theorem for all $k \in K_F$. The domains of the subproblems for commodities $k \in K_F$ are given by the polytopes:

$$\mathcal{P}_k = \left\{ \begin{array}{lll} \sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} & = & 1 \quad i = O_k \\ \sum_{j \in N} x_{ji} - \sum_{j \in N} x_{ij} & = & 1 \quad i = D_k \\ \sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} & = & 0 \quad i \in N \setminus \{O_k, D_k\} \\ x_{ij} \geq 0 & & (i, j) \in A \end{array} \right\}$$

The convexity constraints for the individual subproblems (3.12) bound the flow between the (O_k, D_k) pair from above (a maximal flow of d_k is possible). The extreme points of \mathcal{P}_k are paths between the (O_k, D_k) pair. Decomposing the arc flow model to the path flow model reduces the size of the constraint set to $|A| + |K| + |N|$, but the number of variables are the extreme points of $\mathcal{P}_k, \forall k \in K_F$, which may be exponential.

The exponential number of variables is handled by considering only a small subset in a restricted master problem. Paths/columns are generated on the fly using delayed column generation. The dual variables corresponding to the three constraint sets are:

- w_{ij} for each $(i, j) \in A$ corresponding to the bundle constraints (3.11)
- σ^k corresponding to the convexity constraints (3.12) for each commodity $k \in K$
- α^i corresponding to the inter-balancing constraints (3.13) for each port $i \in N$.

The reduced cost \hat{c} of a path $p \in P_k$ for the variable $f(p)$ in subproblem $k \in K_F$ is given as:

$$\hat{c}_p = r_k - \sum_{(i,j) \in A} \delta_{ij}(p) (c_{ij}^k - w_{ij}) - \sigma^k - \alpha^{O_k} + \alpha^{D_k}$$

Note that the dual values α^i cancel each other out for intermediate ports on a path in constraints (3.13), i.e., only the supply port O_k and the demand port D_k of a path are affected by these constraints.

As the subproblems are only dependent on the arc variables x_{ij} , the constant terms given by commodity k may be treated independently implying:

$$\sum_{(i,j) \in A} \delta_{ij}(p)(c_{ij}^k + w_{ij}) < r_k - \sigma^k - \alpha^{O_k} + \alpha^{D_k}$$

An extreme point of \mathcal{P}_k where $\sum_{(i,j) \in A} (c_{ij} + w_{ij})x_{ij}$ is minimal corresponds to the path variable with the maximal reduced cost for the subproblem belonging to k . The subproblem corresponds to an ordinary shortest path problem with positive arc costs as $c_{ij}, w_{ij} \geq 0$:

$$\min \sum_{(i,j) \in A} (c_{ij} + w_{ij})x_{ij} \quad (3.18)$$

$$\text{s.t. } \sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = 1 \quad i = O_K \quad (3.19)$$

$$\sum_{j \in N} x_{ji} - \sum_{j \in N} x_{ij} = 1 \quad i = D_K \quad (3.20)$$

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = 0 \quad i \in N \setminus \{O_K, D_K\} \quad (3.21)$$

$$x_{ij} \in \{0, 1\} \quad (i, j) \in A \quad (3.22)$$

Solving the LP relaxed path flow model using delayed column generation has several advantages compared to solving the arc flow model:

- Although there is a polynomial bound on the number of variables in the arc flow formulation, it is a large polynomial factor. Even though the number of paths in the network *may* be exponential, it depends on how dense the network is. If the network is sparse, like in most liner shipping networks, there will be few path variables for each commodity $k \in K_F$.
- The network size of an international liner shipping company means that column generation is our only hope to solve the problem in reasonable time. Even the LP relaxation of the arc flow model cannot be represented with 8 GB memory for sufficiently large instances.
- An LP-solution to the path flow formulation can be transformed to a feasible IP solution to CAPER by rounding, as flow conservation is respected implicitly in the path variables (see figures 3.2 and 3.3). This makes it possible to translate the solution directly into itineraries for the demand pairs.
- The average volume of demand between (O, D) pairs are in 3 orders of magnitude. As a result rounding down the fractional solution is a reasonable approximation to the optimal integer solution. This is confirmed by the experimental results of this paper, where the gap between the integer solution found by rounding and the optimal LP solution is within 0.01%.

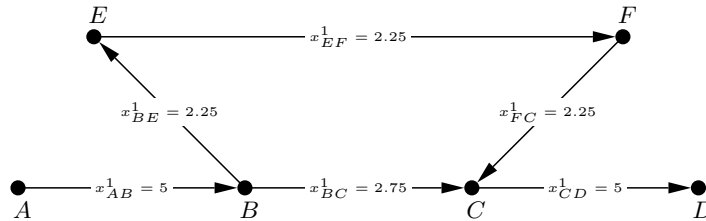


Figure 3.2: A fractional solution to the arc flow model: Containers may be split at every node.

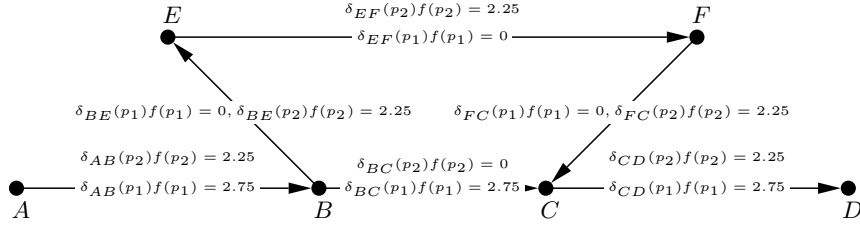


Figure 3.3: Fractional solution to the path flow model: Containers may only be split at the origin.

3.5.2 Solution method

Although the integer formulation of CAPER is \mathcal{NP} -hard to solve, the LP relaxation may be solved in polynomial time. The challenge lies in the expected size of a liner shipping network and the number of periods in the planning horizon, which result in very large LPs.

The solution method for solving the LP relaxed arc flow model is an interior point method using a standard solver. The path flow model is solved using delayed column generation (Wolsey, 1998). Due to load rejection a feasible solution to CAPER is to provide a single path variable for each commodity by solving a shortest path problem for each commodity using Dijkstra on a graph with the original arc cost $c_{ij}^k, \forall (i, j) \in A$. The LP is solved to optimality with an associated dual cost vector. Hereafter, $|K|$ subproblems are solved by applying Dijkstra's shortest path algorithm to each subproblem with arc costs corresponding to the original arc cost and the dual cost of each capacity constraint. The new columns generated by the subproblems are added to the master problem. After resolving the master, arc weights are adjusted and the subproblems are solved again. The column generation algorithm terminates, when no path variable with positive reduced cost can be found for any commodity. The column generation algorithm applied is simple and no improvement methods such as constraint aggregation, non-basic variable removal, etc. is applied to give a fair comparison to solving the arc flow model by a standard solver.

An integer solution is found by applying a rounding heuristic to the optimal LP solution of the path flow model. The rounding heuristic is simple. Initially all fractional variables of the path flow LP relaxation solution are rounded down to ensure that capacity constraints are respected. At nodes with violated inter-balancing constraints we supply empty containers through the leasing variables to maintain feasibility. The rounding heuristic is applied in order to verify that the LP relaxation solution is equitable for evaluating the liner shipping network (see Section 3.6.3).

3.6 Computational Results

The algorithms were experimentally evaluated on data based on real life shipping networks. The test instances are created from a snapshot of the Containership Databank (Containership Databank). The set P of ports (and hence the set N of nodes) and the set of arcs A with capacities are created from services found at (Containership Databank). The time frame of the initial time-space networks are one, three, six, nine and twelve months which are aggregated into one, three, six, nine and twelve time slots respectively representing one month each. Cost and demand functions are generated randomly but such that both profitable and unprofitable products are present, demands are asymmetric in the sense that an area such as Asia should have more export than import (and vice versa for, e.g., Europe and North America), the total demand must exceed the capacity of the network for some areas, and the profit of some products must be able to support the price of leasing containers. Liner shipping operators are chosen so that the instances vary in size from 34 ships to 316 ships. Test instances are named according to the number of ships in the fleet, see Table 3.3. Note that instance CAPER293 is larger than instance CAPER316 in terms of the number of ports and unique rotation legs. In Table 3.3 the average out degree of the nodes in the liner shipping network is stated. The density of the network is believed to be decisive for the convergence of the delayed column generation. The size of the network and the number of

Test instance	Ports	Unique rotation legs	Average out degree	Fleet capacity in TEU
CAPER34	44	101	2.3	21035
CAPER62	60	104	1.7	111004
CAPER98	58	122	2.1	348356
CAPER136	96	198	2.1	383179
CAPER222	151	326	2.2	633719
CAPER293	234	565	2.4	846447
CAPER316	185	455	2.5	992479

Table 3.3: Test instances - instance name denotes the fleet size, e.g., CAPER34 has a fleet size of 34 ships. *Unique rotation leg* is the number of travel arcs A_R disregarding travel times. Instances may be found at <http://www.or.man.dtu.dk/English/research/instances/>

$ T $	instance	$ N $	$ A $	$ K_F $
1	CAPER34	44	101	215
	CAPER62	60	104	342
	CAPER98	58	122	494
	CAPER136	96	198	837
	CAPER222	151	326	1277
	CAPER293	233	565	2189
	CAPER316	185	455	1930
3	CAPER34	132	391	645
	CAPER62	180	432	1159
	CAPER98	174	482	1751
	CAPER136	288	786	2856
	CAPER222	453	1280	4577
	CAPER293	699	2161	7522
	CAPER316	555	1735	6895
6	CAPER34	264	826	1290
	CAPER62	360	924	2515
	CAPER98	348	1022	3865
	CAPER136	576	1688	6163
	CAPER222	906	2711	10301
	CAPER293	1398	4555	16278
	CAPER316	1110	3655	15382
9	CAPER34	396	1261	1935
	CAPER62	540	1416	3950
	CAPER98	522	1562	6081
	CAPER136	864	2550	9606
	CAPER222	1359	4142	16311
	CAPER293	2097	6949	25207
	CAPER316	1665	5575	24404
12	CAPER34	528	1696	2580
	CAPER62	720	1908	4979
	CAPER98	696	2102	7444
	CAPER136	1152	3432	13097
	CAPER222	1812	5573	22433
	CAPER293	2796	9343	34163
	CAPER316	2220	7495	33654

Table 3.4: Network sizes of the respective instances divided into time periods

commodities is stated separately for each time period in table 3.4. The experimental tests are based on the models defining only an in-leasing variable and all nodes in the first time period are given 100 own containers. In a real setting the nodes in the first time period must be supplied with the number of empty stock at each port, to model the liner shipping companies own containers.

All tests were performed on an *Intel(R) Xeon(R) X5550 CPU 2.66 GHz processor* with 8 GB RAM. We have used the LP solvers from *ILOG's CPLEX 10.2* and the open source *CLP* solver from *COIN-OR*. All tests were performed with the *CPLEX* Barrier solver, *CPLEX* dual simplex solver and the *CLP* dual simplex solver. The best results for the arc flow model were obtained with *CPLEX* barrier solver and for the path flow model with *CLP* dual simplex. Computational results are stated for the best results for each model for 1, 3, 6, 9, and 12 time periods. The models are solved to LP-optimality. The arc flow model of large instances cannot be generated with the available memory.

In the following we compare performance of the arc flow and the path flow model for test instances with 1 and 3 time periods, where the arc flow model can be generated for most instances. Next we present results for the solution times for large instances using the path flow model in conjunction with delayed column generation. For the path flow model all test instances up to

$ T $	instance	objective	arc flow model			path flow model					
			m	\times	n	time(s)	m	\times	n	time(s)	iter
1	CAPER34	$2.59 \cdot 10^{07}$	21860	\times	9605	4.76	360	\times	378	0.02	4
	CAPER62	$1.21 \cdot 10^{09}$	35732	\times	20684	23.30	506	\times	542	0.03	4
	CAPER98	$5.37 \cdot 10^{09}$	60448	\times	28832	46.40	674	\times	889	0.06	4
	CAPER136	$4.32 \cdot 10^{09}$	166020	\times	80646	180.00	1131	\times	1589	0.20	6
	CAPER222	$7.92 \cdot 10^{09}$	416779	\times	193304	952.00	1754	\times	2285	0.34	5
	CAPER293	$1.1 \cdot 10^{10}$	1237583	\times	510835	1670.00	2987	\times	4008	1.03	8
	CAPER316	$1.22 \cdot 10^{10}$	878790	\times	357690	1140.00	2570	\times	3352	0.70	6
3	CAPER34	$7.9 \cdot 10^{07}$	252718	\times	85663	580.00	1168	\times	2962	0.40	5
	CAPER62	$4.3 \cdot 10^{09}$	501300	\times	209232	1730.00	1771	\times	2159	0.63	10
	CAPER98	$1.98 \cdot 10^{10}$	844638	\times	305330	5420.00	2407	\times	3638	1.73	15
	CAPER136	$1.55 \cdot 10^{10}$	2245890	\times	823602	5690.00	3930	\times	5863	2.98	11
	CAPER222	$2.62 \cdot 10^{10}$	5860293	\times	2075114	57600.00	6310	\times	10039	14.10	16
	CAPER293	$3.65 \cdot 10^{10}$	16257902	\times	5260738	MEM	10382	\times	16620	38.20	14
	CAPER316	$5.06 \cdot 10^{10}$	11965115	\times	3829015	MEM	9185	\times	14097	33.50	24

Table 3.5: Results for solving test instances for 1 and 3 time periods ($|T|$). The size of the respective models is stated as $m \times n$, where m is the number of columns(variables) and n the number of rows(constraints). MEM indicates that memory was not sufficient for the process to complete. The size of the LP for the arc flow model is reported for comparison with the size of the master problem of the path flow model. Column **time(s)** denotes the CPU time in seconds to solve the respective model, while **iter** denotes the number of iterations for the column generation algorithm. The arc flow model is solved in one LP iteration.

9 time periods can be solved within an hour. For 12 time periods, 5 out of 7 instances may be generated with the available memory. They are solved in less than 71 minutes. Lastly, we present the integer solutions for a simple rounding heuristic applied to the LP solutions of the path flow model. The IP solutions presented are within a very reasonable distance of the LP upper bound and the gap is sufficiently small to discard the need for a branch-and-price algorithm as well as more sophisticated heuristics.

3.6.1 Arc flow and path flow compared

Table 3.5 shows the relative performance of the arc flow model and the path flow model for 1 and 3 time periods respectively. The path flow formulation in conjunction with delayed column generation outperforms the arc flow model by a wide margin even for small instances. The size of the LPs for the column generation algorithm is surprisingly small and the column generation algorithm is at least two orders of magnitude faster than the arc flow model for one time period and three orders of magnitude faster for three time periods.

Figures 3.4(a)–3.4(b) plot the solution time of the algorithms as a function of the size of the LP model. The unit $|N| + |A| + |K|$ is chosen at the x -axis because it is the decisive factors in the size of the LP constraint set for both models. Figure 3.4(a) shows a fast growth in solution time for the arc flow model (full lines). Using a logarithmic scale in Figure 3.4(b) it is seen that the original growth is exponential. The solution times of the path flow model (dotted lines) grow more moderately. Figures 3.4(c)–3.4(d) correspond to figures 3.4(a)–3.4(b) when considering three time periods. Again, Figure 3.4(d) shows an exponential growth in the solution time of the arc flow model (the two largest instances could not be generated due to memory limitations).

3.6.2 Increasing $|A|$

It is believed that the relatively fast convergence of the path flow model even for large instances is due to the very sparse networks in our real life like instances. The number of extreme points in the subproblems is therefore very small and yields small *master problems*. This means that only few iterations are needed to find all relevant basic variables via the column generation method. A test has been performed to support this theory. Test case CAPER62 was chosen randomly to perform this test. In three steps the graph was made increasingly dense to see how this would effect execution time. However, density is performed in a way that creates alternative paths for existing commodities. In the tests the same node set and commodity set is used. Only the rotations are

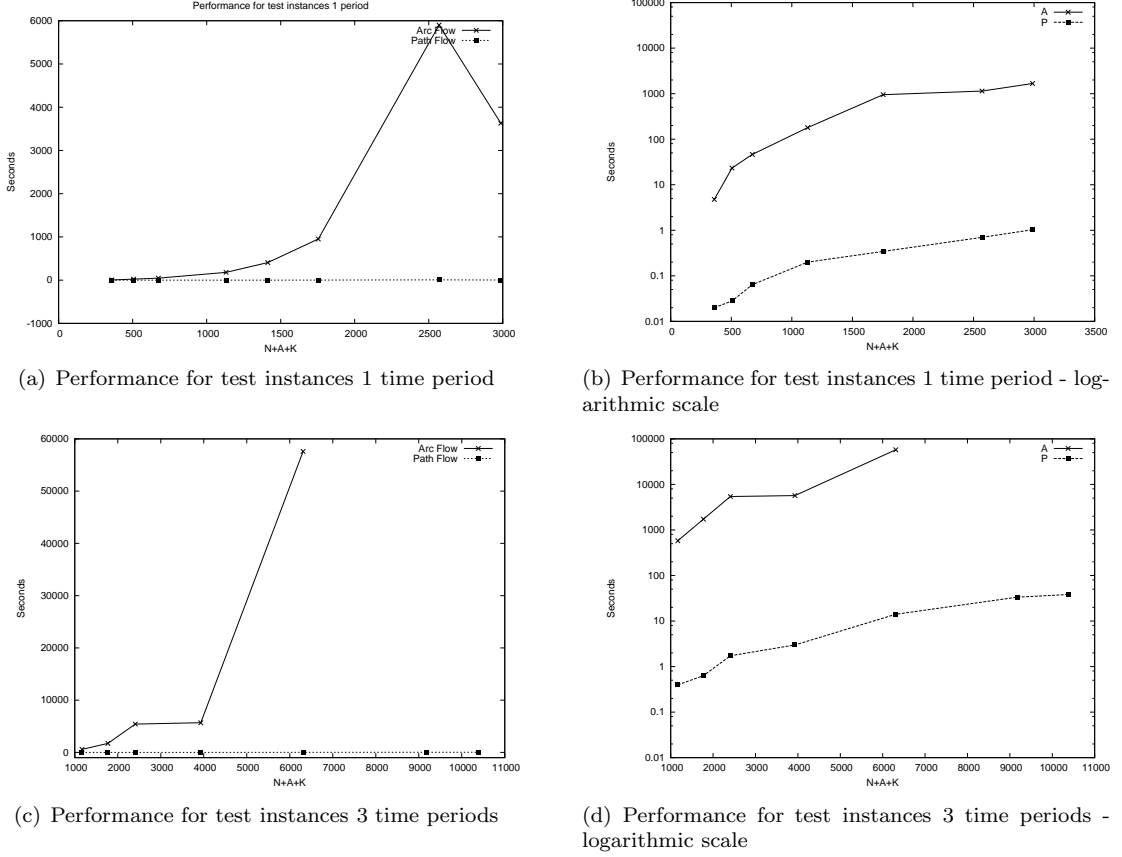


Figure 3.4: Relative performance of the arc flow and path flow model, 1 and 3 time periods. The size of the test instance in terms of the accumulated number of nodes $|N|$, arcs $|A|$ and commodities $|K|$ is on the x-axis. The computation time in seconds is on the y-axis.

different. Steps one and two create competitive routes for the existing commodity set while the third step add routes in a random way to the existing commodity set.

Each step was completed as a test in itself. The original run had 421 arcs, the test where only step one was performed has 725 arcs. Performing step one and two on the original test instance yields 860 arcs and completing all three steps results in 1073 arcs. The number of vertices for all test instances is 183. It is noted that the average out degree of the largest arc set is only 5.85, which is still a fairly sparse network.

Figure 3.5 clearly displays a steep rise in execution times for the path flow formulation using delayed column generation when the graph becomes denser. This supports our theory on why our real life like instances are solved fast and do not show signs of degeneracy.

3.6.3 Solution of large instances

We now consider the solution times for 6, 9 and 12 time periods. The arc flow model cannot be generated for the largest instances and hence is not discussed further in this section.

Table 3.6 shows that test instances with 6 time periods solved with the path flow model complete within 10 minutes. The master problems are small compared to the arc flow model and the number of iterations is reasonable. The sparsity of the networks probably results in few path variables for a commodity, which leads to relatively fast convergence of the delayed column generation algorithm. Note that test CAPER316 is slower than test CAPER293 in spite of a smaller LP. This might be specific for test CAPER316 but may also be due to degeneracy, ϵ rounding or many cache misses.

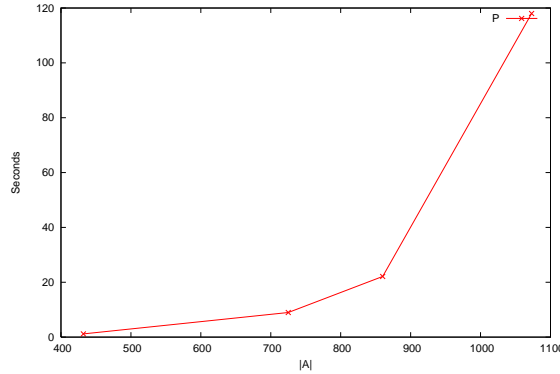


Figure 3.5: Effect of increasing the density of a graph. The number of arcs in the graph $|A|$ is on the x-axis. The computation time in seconds is on the y-axis.

Test instances with 9 time periods may all be completed in less than one hour. All but the two largest tests, CAPER293 and CAPER316 complete within 12 minutes. The number of columns in the two largest test instances is significant. Again we see that test CAPER316 needs more time and iterations to complete than test CAPER293 although the LP is smaller. The increased solution time seems to be problem specific and it is interesting to note that the network of CAPER316 is denser than that of CAPER293. This supports the theory that the sparsity of shipping networks is a key to success for the path flow model and the column generation algorithm. For 9 time periods we see that the number of iterations varies from 9 to 89. However, execution times still grow steadily and the size of the LPs is reasonable considering the size of the networks. For 12 time periods the LPs have reached a critical size and tests CAPER293 and CAPER316 do not have sufficient memory to complete.

The number of iterations for the remaining test instances is reasonable and the solution times are still within 71 minutes which is acceptable for large models representing a shipping network for a whole year.

Figure 3.6(a)-3.6(b) depict the solution time of the path flow model for 1, 3, 6, 9, and 12 time periods ($|T|$) as a function of the size of the LP. It is seen that the growth in solution times is relatively steady for 3 time periods. Figure 3.6(a) shows an exponential tendency for the graphs of 9 and 12 time periods, where the LPs have reached a critical size. The trend is even more explicit in figure 3.6(b) where the graphs are plotted on a logarithmic scale. The trend is particular for the larger test instances, which have denser networks and hence, more path variables per commodity. The exponential tendency is very clear in the graph of 12 time periods although the two largest tests did not complete.

The delayed column generation method shows good convergence for the generated test instances, and methods to reduce the master problem constraint set has not been required. We are able to solve instances of large shipping networks spanning 9 time periods in less than one hour. For 12 time periods the two largest instances cannot be generated with the available memory, but the remaining tests are solved within 71 minutes. The convergence is expected to depend on the sparsity of the networks. The results of the tests show that the column generation technique is very effective for solving CAPER for sparse networks. The path flow model and delayed column generation outperforms solving the arc flow model with an interior point method by a wide margin, contrary to the general view on interior point versus LP column generation for large instances. It appears that the number of path variables for real life liner companies is very modest and therefore the number of variables in the restricted master problem is relatively small. Instances are based on modified real-life data which indicates that CAPER will perform well on such problems, but the actual partition of surplus/deficit zones for empty containers and the commodity set may result in harder instances than the generated instances presented in this paper.

Table 3.7 shows the average utilization of travel arcs ($\text{Av. util } A_R$), the number of arcs that

$ T $	instance	objective	arc flow model				path flow model				
			m	\times	n	time(s)	m	\times	n	time(s)	iter
6	CAPER34	$2.49 \cdot 10^{08}$	1066630	\times	341650	3410.0	2380	\times	11070	4.6	7
	CAPER62	$9.18 \cdot 10^{09}$	2325144	\times	906684	7300.0	3799	\times	5305	5.7	19
	CAPER98	$5.03 \cdot 10^{10}$	3951400	\times	1346390	35100.0	5235	\times	7989	8.6	12
	CAPER136	$3.9 \cdot 10^{10}$	3552132	\times	10282128	MEM	8407	\times	15251	43.7	17
	CAPER222	$7.29 \cdot 10^{10}$	9339332	\times	27929628	MEM	13918	\times	25635	206.0	23
	CAPER293	$7.71 \cdot 10^{10}$	22762597	\times	74147688	MEM	22231	\times	41419	480.0	23
	CAPER316	$1.18 \cdot 10^{11}$	17078785	\times	56225975	MEM	20147	\times	36229	586.0	29
9	CAPER34	$4.98 \cdot 10^{08}$	767917	\times	2441692	12500.0	3592	\times	11205	5.7	9
	CAPER62	$1.78 \cdot 10^{10}$	2134956	\times	5595156	MEM	5906	\times	12790	48.7	89
	CAPER98	$7.11 \cdot 10^{10}$	3176366	\times	9500606	MEM	8165	\times	14201	66.1	21
	CAPER136	$7.17 \cdot 10^{10}$	8302998	\times	24496164	MEM	13020	\times	24212	186.0	41
	CAPER222	$1.26 \cdot 10^{11}$	22172150	\times	67565663	MEM	21812	\times	42444	618.0	17
	CAPER293	$1.37 \cdot 10^{11}$	52866028	\times	175158501	MEM	34252	\times	69529	2010.0	27
	CAPER316	$2.18 \cdot 10^{11}$	17078785	\times	56222321	MEM	31643	\times	63309	3120.0	45
12	CAPER34	$3.69 \cdot 10^{08}$	1364464	\times	4377904	MEM	4804	\times	20557	20.0	5
	CAPER62	$1.4 \cdot 10^{10}$	3587508	\times	9502560	MEM	7607	\times	36750	125.0	27
	CAPER98	$8.62 \cdot 10^{10}$	5183822	\times	15650086	MEM	10242	\times	39919	210.0	18
	CAPER136	$1.24 \cdot 10^{11}$	15092328	\times	44953488	MEM	17681	\times	36455	329.0	30
	CAPER222	$1.72 \cdot 10^{11}$	40655981	\times	125020921	MEM	29818	\times	67841	4240.0	32
	CAPER293	-	95531887	\times	319187701	MEM	46302	\times	114088*	MEM	25*
	CAPER316	-	74721600	\times	252239000	MEM	43369	\times	90759*	MEM	40*

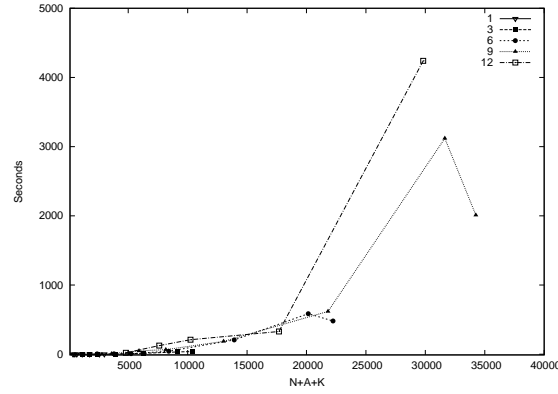
Table 3.6: Test instances for 6, 9 and 12 time periods ($|T|$). The size of the respective models is stated as $m \times n$, where m is the number of columns(variables) and n the number of rows(constraints). MEM indicates that memory was not sufficient for the process to complete. The size of the LP for the arc flow model is reported for comparison with the size of the master problem of the path flow model. Column **time(s)** denotes the CPU time in seconds to solve the respective model, while **iter** denotes the number of iterations for the column generation algorithm. The arc flow model is solved in one LP iteration. * indicates the size of the LP at the last iteration of the process when aborting due to insufficient memory.

are fully loaded, i.e., bottlenecks, per total number of travel arcs ($|BN|/|A_R|$) and the percentage of arcs being bottleneck (BN). Generally, the capacity is well utilized apart from CAPER34. A large fraction of the travel arcs are fully utilized and constitute bottlenecks in the network. In strategic planning, identification of bottlenecks is crucial along with sensitivity analysis when redesigning parts of the network.

3.6.4 Quality of integer solutions and speed of rounding heuristic

Table 3.8 considers the integer solutions obtained by rounding.

Table 3.8 shows that 9 out of 33 ($\approx 27\%$) of the LP solutions are integer. The optimal LP solutions of test instance CAPER34 are integer for all values of $|T|$. We observe that 17 out of 33 ($\approx 51\%$) LP solutions have less than 10% fractional basic variables. The highest percentage of fractional variables among the remaining 16 LP solutions is 21.8%. Fractionality seems to increase with $|T|$. Despite having more than 20% fractional basic variables at most 0.3% of the total flow is rounded and the gap percentage in terms of the objective value does not exceed 0.01%. This confirms that the rounded integer solution is a good solution in terms of the gap to the LP upper bound. This is probably due to generally large flows on path variables making the rounding insignificant. Execution times are mostly less than one second but on larger instances execution times rise to at most 5 seconds. The optimal integer solution might be slightly better, but given a gap less than $10^{-4}\%$ the computation time does not seem justified since the gap is smaller than



(a) Performance of model P for 3, 6, 9, and 12 time periods

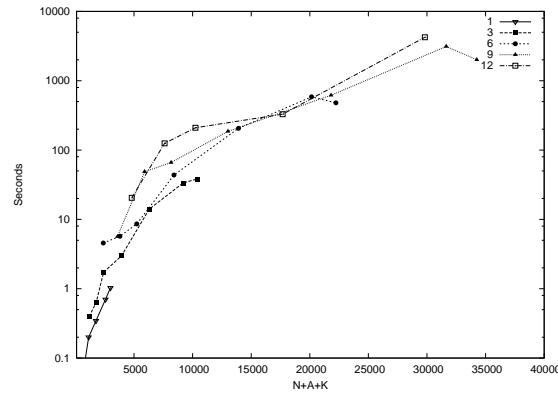
(b) Performance of model P for 3, 6, 9, and 12 time periods
- logarithmic scale

Figure 3.6: Relative performance of the path flow model - 1,3,6,9 and 12 time periods The size of the test instance in terms of the accumulated number of nodes $|N|$, arcs $|A|$ and commodities $|K|$ is on the x-axis. The computation time in seconds is on the y-axis.

the data uncertainty.

3.7 Concluding Remarks

We have presented a mathematical model for the cargo allocation problem with empty repositioning and solved it to near-optimality using delayed column generation and a simple rounding heuristic. CAPER resembles the model of Erera et al. (2005) in managing loaded and empty containers simultaneously. However, CAPER is solved for a capacitated network, allows load rejection and has a profit maximizing objective which in our opinion creates a novel cargo allocation model. Furthermore, CAPER is decomposed and solved by delayed column generation. The method is computationally tractable for much larger LPs than presented in Erera et al. (2005). The mathematical model is surprisingly simple. The inter-balancing constraints, which ensure repositioning of empty containers, results in an augmented multi-commodity flow problem. It appears that these constraints do not complicate the model to an extent where the solution time is significantly affected. Furthermore, test results show that the inter-balancing constraints ensure transportation of low profitable products before repositioning empty containers to deficit ports, see Løfstedt (2007) for a detailed discussion. This demonstrates the importance of considering empty container repositioning in a cargo allocation/booking model for liner shipping. Also, the size of the instances created and solved in this paper are significantly larger than previously reported in the reviewed

$ T $	instance	Av. util A_R (%)	$ BN / A_R $	BN (%)
1	CAPER34	58.8	22/101	21.8
	CAPER62	80.7	51/104	49.0
	CAPER98	94.6	76/122	62.2
	CAPER136	84.3	97/198	49.0
	CAPER222	84.7	162/326	49.7
	CAPER293	82.9	294/565	52.0
	CAPER316	88.1	259/455	56.9
3	CAPER34	56.6	69/303	22.8
	CAPER62	82.3	162/342	47.4
	CAPER98	91.9	255/365	69.9
	CAPER136	85.5	382/594	64.3
	CAPER222	87.4	632/978	64.6
	CAPER293	82.0	1021/1694	60.3
	CAPER316	90.8	978/1365	71.6
6	CAPER34	60.6	149/606	24.6
	CAPER62	83.9	391/624	62.7
	CAPER98	91.7	551/732	75.3
	CAPER136	87.9	853/1188	71.7
	CAPER222	88.4	1375/1956	70.3
	CAPER293	80.3	2052/3385	60.6
	CAPER316	91.8	2126/2730	77.9
9	CAPER34	56.5	206/909	22.7
	CAPER62	83.3	589/936	62.9
	CAPER98	90.1	817/1098	74.4
	CAPER136	85.0	1135/1782	63.7
	CAPER222	87.3	2031/2934	69.2
	CAPER293	82.5	3293/ 5085	64.8
	CAPER316	91.6	3169/ 4095	77.4
12	CAPER34	47.2	209/1212	17.2
	CAPER62	85.5	794/1248	63.6
	CAPER98	88.8	1051/1464	71.8
	CAPER136	85.9	1606/2376	67.6
	CAPER222	87.4	2748/ 3912	70.2
	CAPER293	-	-	-
	CAPER316	-	-	-

Table 3.7: Average utilization of travel arcs in the network $Av. \text{ util } A_R$. $BN \subseteq A_R$ denotes the fully loaded arcs, i.e., bottlenecks, in the network. $|BN|/|A_R|$ is the relation between the number of bottleneck travel arcs and $|A_R|$ also provided in percentage in the last column.

literature. The cargo allocation problem with empty repositioning may aid network designers in evaluating their network from a network pricing perspective. Changes to the network may be evaluated with regards to the effect of empty repositioning and overall cargo flow. The problem may also aid revenue managers in their pricing decisions as the derived empty repositioning cost of each commodity may be incorporated into overall transportation cost.

Solving the LP relaxed path flow model with delayed column generation turned out to be very successful compared to solving the arc flow model with the *CPLEX* barrier solver. The column generation algorithm is at least two orders of magnitude faster for one time period and three orders of magnitude faster for three time periods. The column generation algorithm is able to solve all instances for 6 time periods in 586 seconds. For 9 time periods test instance 316 containing 1110 nodes (185 ports in 9 periods), 3655 arcs and 15382 commodities is solved in 3120 seconds. For 12 periods the two largest test instances cannot be solved with the available memory. The largest instance completed contains 1812 nodes (222 ports in 12 periods), 5573 arcs and 22433 commodities and is solved in 4240 seconds. A rounding heuristic is applied to the LP solutions of the path flow model with great success. The heuristic finds a solution in less than 5 seconds. All integer solutions have a gap to the LP upper bound of at most 0.01% which is well below the data uncertainty.

An area of future work is to incorporate booking and empty repositioning into routing/scheduling decisions of the vessel fleet as the overall cost of running a liner shipping company is the fixed cost of committing to a schedule. Also, it would be very interesting to incorporate substitution of containers into the path flow model of CAPER. Some modifications may result in hard pricing problems (complexity and computationally wise), but we believe that even a complex pricing problem may be solved in reasonable time as the graph may be split into sub graphs according to time periods, and because paths are generally very short. Furthermore, pricing problems may be solved

$ T $	instance	fractional/basic	fractional (%)	Rounded (%)	obj gap	gap (%)	time (s)
1	CAPER34	0/183	0.0	0.0	0	0.0	0.01
	CAPER62	0/214	0.0	0.0	0	0.0	0.01
	CAPER98	0/331	0.0	0.0	0	0.0	0.01
	CAPER136	10/538	1.9	$2.7 \cdot 10^{-4}$	$1.4 \cdot 10^4$	$3.2 \cdot 10^{-6}$	0.01
	CAPER222	0/836	0.0	0.0	0	0.0	0.02
	CAPER293	23/1430	1.6	$2.2 \cdot 10^{-4}$	$2.9 \cdot 10^3$	$2.7 \cdot 10^{-6}$	0.06
	CAPER316	28/1210	2.3	$2.5 \cdot 10^{-4}$	$3.4 \cdot 10^4$	$2.8 \cdot 10^{-6}$	0.05
3	CAPER34	0/588	0.0	0.0	0	0.0	0.02
	CAPER62	46/723	6.4	$1.4 \cdot 10^{-3}$	$8.2 \cdot 10^4$	$1.9 \cdot 10^{-6}$	0.03
	CAPER98	116/1020	11.4	$1.06 \cdot 10^{-3}$	$2.7 \cdot 10^5$	$1.37 \cdot 10^{-5}$	0.03
	CAPER136	0/1680	0.0	0.0	0	0.0	0.08
	CAPER222	324/2600	12.5	$1.81 \cdot 10^{-3}$	$7.7 \cdot 10^5$	$2.93 \cdot 10^{-5}$	0.20
	CAPER293	375/4450	8.4	$1.41 \cdot 10^{-3}$	$9 \cdot 10^5$	$2.47 \cdot 10^{-5}$	0.48
	CAPER316	161/3620	4.4	$5.03 \cdot 10^{-4}$	$5.2 \cdot 10^5$	$1.02 \cdot 10^{-5}$	0.36
6	CAPER34	0/1060	0.0	0.0	0	0.0	0.04
	CAPER62	102/1330	7.7	$1.7 \cdot 10^{-3}$	$4.6 \cdot 10^5$	$4.98 \cdot 10^{-5}$	0.10
	CAPER98	290/1960	14.8	$1.37 \cdot 10^{-3}$	$1.3 \cdot 10^6$	$2.66 \cdot 10^{-5}$	0.14
	CAPER136	453/3290	13.8	$2.17 \cdot 10^{-3}$	$1.6 \cdot 10^6$	$4.13 \cdot 10^{-5}$	0.36
	CAPER222	969/5060	19.1	$2.40 \cdot 10^{-3}$	$3.8 \cdot 10^6$	$5.28 \cdot 10^{-5}$	0.86
	CAPER293	1114/9460	11.8	$2.14 \cdot 10^{-3}$	$3.7 \cdot 10^6$	$4.74 \cdot 10^{-5}$	2.01
	CAPER316	1281/7660	16.7	$2.04 \cdot 10^{-3}$	$5.2 \cdot 10^6$	$4.44 \cdot 10^{-5}$	1.55
9	CAPER34	0/1560	0.0	0.0	0	0.0	0.10
	CAPER62	259/1990	13.0	$3.22 \cdot 10^{-3}$	$1.8 \cdot 10^6$	$1.02 \cdot 10^{-4}$	0.21
	CAPER98	673/3080	21.8	$2.18 \cdot 10^{-3}$	$3.8 \cdot 10^6$	$5.30 \cdot 10^{-5}$	0.29
	CAPER136	744/4800	15.5	$2.44 \cdot 10^{-3}$	$4.3 \cdot 10^6$	$5.94 \cdot 10^{-5}$	0.82
	CAPER222	1568/7610	20.6	$3.05 \cdot 10^{-3}$	$8.1 \cdot 10^6$	$6.41 \cdot 10^{-5}$	1.99
	CAPER293	1838/13900	13.2	$2.39 \cdot 10^{-3}$	$7.5 \cdot 10^6$	$5.49 \cdot 10^{-5}$	4.93
	CAPER316	2357/11500	20.5	$2.61 \cdot 10^{-3}$	$1.3 \cdot 10^7$	$5.92 \cdot 10^{-5}$	3.70
12	CAPER34	0/2030	0.0	0.0	0	0.0	0.15
	CAPER62	50/2470	2.0	$4.53 \cdot 10^{-4}$	$3.7 \cdot 10^5$	$2.61 \cdot 10^{-5}$	0.38
	CAPER98	532/3300	16.1	$1.25 \cdot 10^{-3}$	$2.9 \cdot 10^6$	$3.32 \cdot 10^{-5}$	0.49
	CAPER136	630/6280	10.0	$1.63 \cdot 10^{-3}$	$5.5 \cdot 10^6$	$4.41 \cdot 10^{-5}$	1.46
	CAPER222	2163/10600	20.3	$3.05 \cdot 10^{-3}$	$1.4 \cdot 10^7$	$7.89 \cdot 10^{-5}$	3.59
	CAPER293	-	-	-	-	-	-
	CAPER316	-	-	-	-	-	-

Table 3.8: Rounded integer solutions presented as the fraction of non-integer basic variables (**fractional/basic**), the percentage of non-integer basic variables (**fractional**), the fractional moves as a percentage of total moves (**Rounded**), the objective gap (**obj gap**) and gap percentage (**gap**) between the LP and IP solution and finally, the CPU time for the rounding heuristic in seconds (**time**)

in parallel to decrease solution times. For network evaluation it might prove significant to evaluate the competitiveness of the network in terms of e.g. transit time of a commodity. Reinhardt and Pisinger (2011) solve a multi-objective shortest path problem for liner shipping with non-additive costs. These techniques could be relevant for CAPER because it is likely that several criteria need to be taken into account when defining the attractiveness of a path and various strategic goals may have a non-additive cost structure.

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Chapter 4

The Vessel Schedule Recovery Problem (VSRP) - a MIP model for handling disruptions in liner shipping

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Abstract Containerized transport by liner shipping companies is a multi billion dollar industry carrying a major part of the world trade between suppliers and customers. The liner shipping industry has come under stress in the last few years due to the economic crisis, increasing fuel costs, and capacity outgrowing demand. The push to reduce CO_2 emissions and costs have increasingly committed liner shipping to *slow-steaming policies*. This increased focus on fuel consumption, has illuminated the huge impacts of operational disruptions in liner shipping on both costs and delayed cargo. Disruptions can occur due to adverse weather conditions, port contingencies, and many other issues. A common scenario for recovering a schedule is to either increase the speed at the cost of a significant increase in the fuel consumption *or* delaying cargo. Advanced recovery options might exist by swapping two port calls or even omitting one. We present the Vessel Schedule Recovery Problem (VSRP) to evaluate a given disruption scenario and to select a recovery action balancing the trade off between increased bunker consumption and the impact on cargo in the remaining network and the customer service level. It is proven that the VSRP is \mathcal{NP} -hard. The model is applied to four real life cases from Maersk Line and results are achieved in less than 5 seconds with solutions comparable or superior to those chosen by operations managers in real life. Cost savings of up to 58% may be achieved by the suggested solutions compared to realized recoveries of the real life cases.

Keywords: disruption management, liner shipping, mathematical programming, recovery

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4.1 Introduction

Disruptions occur often in a global liner shipping network. According to Notteboom (2006) approximately 70-80% of vessel round trips experience delays in at least one port. The common causes are bad weather, strikes in ports, congestions in passageways and ports, and mechanical failures. More exceptional causes include piracy and crew strikes on the vessels.

Example: The vessel Maersk Sarnia is deployed on a scheduled service providing transport of container cargo between South-East Asia and the west coast of Central America, see Figure 4.1. During the pickup of cargo in South-East Asia the weather conditions cause Maersk Sarnia to suffer a 30 hour delay when leaving Kwangyang in South Korea. The delay can cause the vessel to miss an important scheduled port call in the transshipment port of Balboa in Panama. As a result large parts of the cargo will miss their onward connections and most cargo will not be delivered on time.

In order to mitigate the negative effects of the delay on Maersk Sarnia the operations center at Maersk Line has several options:

- Omit the upcoming port calls at Yokohama, Lazaro Cardenas, or Balboa.
- Speed up significantly to try to reach Balboa on time.
- Swap the port calls of Lazaro Cardenas and Balboa.
- Accept the delay and catch up the schedule returning to South-East Asia from Balboa.

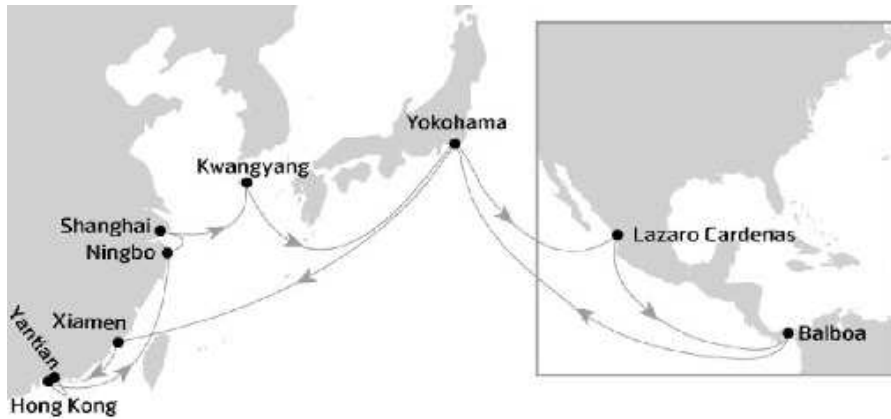


Figure 4.1: A trans-Pacific round trip is depicted. Cargo is collected in transshipment ports in Asia and sailed to transshipment ports in Central America. The round trip takes 56 days implying that 8 vessels is required to maintain a weekly service. Feeder vessels are used to connect all ports in a geographical area.

Currently when a disruption occur, the operator at the shipping companies manually decides what action to take. For a single delayed vessel a simple approach could be to speed up. However, the cost of bunker fuel is a cubic function of speed (Alderton, 2004) and vessels' speeds are limited between a lower and upper limit. So even though an expensive speed increase strategy is chosen, a vessel can arrive late for connections, propagation delays to other parts of the network.

In recent years liner companies have had an increased focus on minimizing the bunker consumption in order to provide environmentally friendly transport and to minimize the operational costs (Maersk, 2010). On the other hand, on time delivery is very important for a global liner shipping company as delayed cargo carries a high cost by customers and key clients. Nevertheless, the negative effects of miss-connections or delaying a key clients merchandise can be hard to measure against a concrete cost of for example bunker. Furthermore, the ripple effect of the recovery on to the remaining network is very complex to overview for a human. In the considered example Maersk Sarnia recovered the situation by a general speed increase with a high bunker cost, but



Figure 4.2: A feeder service collect containers in the hub in Bremerhaven and transport them to their destinations in Norway.

nevertheless the speed increase did not ensure timely delivery of containers to the hub port of Balboa, and final recovery was done returning to Asia. As a result all the cargo was delayed and some cargo missed the onward connection at the hub. The mathematical model presented in this paper suggested omitting the last port call in Asia reaching the transshipment port without increasing the vessel speed and on time. The cost saving, including a delay penalty, of the suggested solution is more than 20 %.

A standardized way of handling disruptions based on mathematical grounded decision support may significantly lower the cost of handling disruptions as seen in the airline industry (Rakshit et al., 1996; Yu et al., 2003) and simplify implementation of strategic decisions among stakeholders. According to UNCTAD (2010) *slow-steaming* has resulted in a significant increase in delays and they expect carriers to resume higher speeds in order to increase reliability and productivity. According to Notteboom (2006) reliability is generally achieved by introducing sufficient buffer time into a service. We believe that a mathematical decision support tool as the one presented in this paper may result in sustaining a *slow-steaming* policy, while increasing reliability of service without the need to introduce additional buffer time. In this paper, we introduce a mathematical model for handling the most common disruptions in liner shipping called the Vessel Schedule Recovery Problem, VSRP.

We make four contributions: First, we propose a novel formulation for the VSRP inspired by similar models within the airline industry. To the best of our knowledge the present article is the first to apply optimization to handle disruption management within the domain of liner shipping networks. Secondly, We prove the VSRP to be NP-complete. Third, we report computational results for four cases representing common disruptions, selected by experienced personnel at Maersk Line Operations Center. The recovery options identified by the mathematical model are comparable or superior to the decisions implemented in real life with cost savings of as much as 58%. The model is solved by a MIP solver within seconds for the selected cases. Fourth, a set of generic test instances is used to provide insights into the network sizes that may be handled in seconds by the current model and solution methods.

The remainder of the paper is organized as follows. Section 4.2 introduces disruption management in the liner shipping business. Section 4.3 describes related literature. In Section 4.4 we introduce the *Vessel Schedule Recovery Problem (VSRP)*, the graph topology, and a mathematical model for the VSRP along with proofs of the NP-completeness of the problem. In Section 4.5 we introduce the four real life cases and the generic test instances and report computational results. Following this section we conclude that a decision support tool based on mathematical optimization of a disruption scenario could greatly aid an operations manager in evaluating the different recovery options.

4.2 The Liner Shipping Business

Liner shipping of containers is the backbone of world trade. Even though containerization simplifies the operations and reduces the cost per transported unit, the earned return is less than 10% on assets (Stopford, 2009). Customers demand fast and reliable delivery, while the shipping companies constantly search to cut costs. These issues have motivated major investments in improving the daily operations at large shipping companies (Notteboom, 2006). The liner shipping company referred to as a *carrier* has a public schedule of services. A service consists of a cyclic route with a scheduled time for each port call en route. Containers travel through the network as passengers in a public transit network, often combining several services. The port calls of a service, must usually happen at a predefined time and place in the port, often called the *berth slot*. This is defined by the physical place that the vessels moors, the berth, and a time window where the vessel is serviced. Most carriers provide weekly frequency of port calls. In recent years major companies are using slow-steaming to lower the variable cost and the CO₂ emission (Løfstedt et al., 2011; Rosenthal, 2010; Maersk, 2010). To stay competitive, research has been focused on designing the network to operate as efficiently as possible. For shipping companies, a division of the ports into *hubs and spokes* is common (Christiansen et al., 2007). The network is not a traditional *hub and spoke network design* with direct links between two hubs or a spoke and a hub. As an alternative large vessels operate *main lines* between a set of hub ports and smaller vessels operate *feeder lines* connecting a set of spokes to a hub. An example of a *main line* service between hub ports is given in Figure 4.1 and an example of a *feeder* service servicing a hub and several spokes is given in Figure 4.2.

The motivation for this hub-and-spoke network design is to benefit from the economies of scale on container vessels (Stopford, 2009). The majority of containers are transhipped at least once during transport adding to the operational complexity and the impact of a disruption. Liner shipping companies operate with a head haul and a back haul direction. In the head haul direction vessels are almost full as opposed to the back haul direction. The head haul generally generates the majority of the revenue retrieved by operating the full service. As described above disruptions are accounted for and handled in the network by adding buffer time. Customer demand for fast delivery results in increased speeds and nearly no buffer time on the head haul, whereas the back haul is slower and has more buffer time. Due to the complexity of recovering from a disruption additional buffer time is included on the back haul with the option of a slight speed increase to catch up with the schedule on the back haul.

The most important variable costs in a liner shipping network is the bunker cost, the cost of using passageways such as the Suez and Panama canals, and the cost of calling ports to load and unload cargo. The fixed cost of operating a network in terms of asset costs on vessels, containers, and equipment are significant. Whenever a vessel fails to operate in accordance with the original schedule it is hurting the shipping company's business (and the business of their customers) (Notteboom, 2006). The utilization of vessels will often be affected negatively as containers miss-connect, resulting in a higher cost per transported unit. Furthermore, it might be necessary to arrange alternative transport for the miss-connected units also adding to the cost. Finally, the customers demand a reliable service and expect on time delivery. A major concern is therefore how to handle disruptions when they occur.

For larger liner shipping companies the information about disruptions are gathered in the company's Operational Control Center (OCC), from where decisions are also taken with respect to how the disruptions should be handled. Decisions here are taken in real-time and any system to support this process should support real-time decision making. The reason for this is two-fold. 1) Weather is changing quickly in some parts of the world, which may cause a port to close for a period of time. In such a case it is important to make a reasonable quick decision regarding whether the port should be skipped, which typically will lead to a change of course and the possibility of slowing down and saving on bunker fuel. 2) The other and more important reason is that controllers working in the OCC are in some periods faced with the need for taking many decision and evaluating various alternatives. This is where the requirement for a quick response becomes imperative. For this reason controllers at Maersk have stated 10 seconds as a reasonable response

time for a disruption management system.

4.2.1 From airline disruption to liner shipping disruption

Operations research has for many years been applied extensively in the airline industry (Barnhart, 2009). Initially OR was mainly used in the planning phase, but during the last two decades OR has also found its way into the disruption management tools, which are used on the day of operation where the planned schedule is being executed.

This paper focuses on utilizing the findings in disruption management tools for the airline industry in order to construct a mathematical model of the VSRP to handle disruptions in the context of the liner shipping business. The airline and liner shipping businesses have evident similarities, but also some core differences (Christiansen et al., 2004). Larger airlines and larger liner shipping companies both operate a hub and spoke network, where either passengers or containers need to flow from an origin, through one or more hubs to a destination. Here, they need to arrive with the least possible amount of delay. In this way vessels resemble aircraft and containers resemble passengers. While crew recovery is a significant part of disruption management for an airline, this is not the case for a liner shipping company, as crew always follow the vessel and do not have work rules, which significantly limit the utilization of the vessel. Traditional aircraft recovery as described by Thengvall et al. (2001) or Dienst et al. (2012) makes use of 3 recovery techniques: *Delays*, *Swaps* and *Cancellations*. In addition to these techniques Marla et al. (2011) show that a large improvement in the number of passenger miss-connections can be obtained if *speed-changes* are included as a fourth recovery technique. In the following we discuss how each of these techniques can be applied to disruption management in a liner shipping network:

- *Delays*. For an airline the most straight forward way of handling a disruption is to delay flights and let the delays propagate to the subsequent flights of an aircraft. After a number of delay propagations the initial delay will have disappeared due to the fact that the gap between flights is usually a bit longer than the required turn time and most aircrafts are idle over night. For an airline this recovery technique is unfortunately also the one which, when applied alone, often ends up causing a lot of miss-connections (Dienst et al., 2012). In liner shipping it is also possible to delay the departure of a vessel, but port calls do not have additional slack built into them and container vessels are constantly in service, which means that delay propagation will not be able to resolve a disruption on its own. It will need to be combined with some of the techniques presented below in order to have the desired effect of recovering from a disruption.
- *Swaps*. This is a very efficient recovery technique for an airline, as it can be used to eliminate a lot of delay propagation to subsequent flights. Swaps are possible as an aircraft becomes empty after each flight. As a result one aircraft may be substituted for another. Unfortunately, this technique is not applicable to a liner shipping company, as a container vessel servicing a certain service is never empty and it is both extremely costly and time consuming to empty it completely. While vessels cannot be swapped in the VSRP it is for a liner shipping company possible to swap the order in which ports are being visited, whenever these ports are located geographically close to each other.
- *Cancellations*. This technique is usually not preferred in the aircraft recovery problem, but it is an efficient way of recovering, whenever the airline experience large delays or reduced runway capacity. For a liner shipping company this technique is unfortunately not directly applicable as it would interrupt the service operation of the vessel. In the VSRP it is however possible to cancel or omit a port call. In this case containers, which are destined for the omitted port, are then off-loaded at a subsequent nearby port and containers for on-loading in the omitted port are being held for the next vessel on that service, or another service covering the same ports, which often results in a delay of up to a week.
- *Speed changes*. Including speed changes from a network perspective as an integrated part of disruption management turns out to be a very effective way of balancing passenger delays

versus fuel burn for an airline (Marla et al., 2011). This is in spite of the fact that a flight usually can only be sped up with 8-10% compared to its planned speed. For a vessel, which is originally scheduled to sail at a slow steaming speed of e.g. 16-18 knots, it is possible to speed up with 40% to e.g. 22-24 knots. This additional speed flexibility may be promising for the application of this technique in a liner shipping network.

As it is seen there are some clear similarities in the techniques, which can be applied in recovering a disruption in an airline network, and the techniques, which can be applied in recovering a disruption in a liner shipping network. The aircraft swapping technique available to an airline provides increased interaction between aircrafts in an airline network as opposed to vessels in a liner shipping network. An additional complication in a liner shipping network is that vessels operate around the clock and cannot naturally recover by using some of the overnight slack, which is often available in an airline network. For this reason recovering from a liner shipping disruption may take days and even weeks as opposed to a typical maximum of 48 hours for airlines. If a container fails to connect to a succeeding vessel the impact will often be more severe in liner shipping. International airports have a number of daily departures for a given destination presenting the option to re-accommodate passengers with a slight delay on a subsequent flight. For liner shipping a missed connection will normally result in a major delay.

We must estimate the effect on the cargo on-board with regards to missed onward connections and delays in order to assess a given recovery plan. Ideally the container groups would be reflowed on the residual capacity of the entire liner shipping network simultaneously with a recovery plan for the delayed vessels. This would significantly increase the graph of our instance as the containers on-board will include services, not considered in the disruption scenario. Additionally, reflowing the cargo is a large scale multicommodity flow problem. Mathematical models incorporating a large multicommodity flow problem such as capacitated network design (Frangioni and Gendron, 2009) and liner shipping network design (Álvarez, 2009) are severely restrained by the size of the problem and excessive solution times for general MIP solvers. We expect similar issues if incorporating the reflow of miss-connected containers into the VSRP and most certainly the application will no longer be able to provide real time suggestions when considering reflowing containers on the residual capacity of the network in a joint optimization. This is furthermore supported by findings in the airline literature where Bratu and Barnhart (2006) concludes that a combined model for solving a combined aircraft recovery and passenger re-accommodation model is too complex to solve to make it useful for real time optimization. Similarly the review Clausen et al. (2009) shows that full passenger re-accommodation is always handled in a subsequent optimization phase. An approach, which has been useful in the airline industry (Marla et al., 2011) is not to solve the full passenger re-accommodation problem together with aircraft recovery, but rather let the aircraft recovery be guided towards passenger friendly solutions by penalizing misconnecting passengers. A similar approach could be deployed for disrupted containers.

4.3 Literature review

Notteboom (2006) analyze the negative effects of disruptions in liner shipping and the actions taken by liner shipping companies to mitigate them. The recent paper by Notteboom and Vernimmen (2009) demonstrates how the increased bunker price has a significant impact on the liner shipping business. The cost of fuel is a dominant cost driver when transporting containers, nevertheless shipping companies are willing to burn extra fuel to arrive according to the schedule. Disruption management is a major concern for liner shippers given this trade-off. Notteboom and Vernimmen (2009) argue that the increased price on bunker has resulted in lowering the speed of vessels to save fuel, which in turn gives the vessels more buffer time and the operators more possibilities to recover from a disruption.

Even though the research within maritime transportation has gained increased focus during the last decades, we have encountered no journal papers devoted to disruption management in (liner) shipping. This can be caused by various things; firstly as mentioned the usage of mathematical

modeling in maritime transportation is still in its infancy and secondly the market of liner shipping is extremely competitive. The development of decision support software will often be carried out for a major player in the market and therefore not necessarily published. After the submission of this article another model on disruption management in liner shipping was published in the thesis of Kjeldsen (2012). A heuristic is presented for solving a relaxed version of the model and computational results are provided for a set of generated disruption scenarios. The work by Yang et al. (2010) and Li et al. (2009) addresses disruption management for berth allocation in container terminals. Their papers are focused on how to recover the berthing schedules when vessels are delayed from the terminal point of view. Yang et al. (2010) presents an MIP Model and a heuristic solution approach. The problem handled is very different from the VSRP dealing with disruptions from the carriers point of view. The work of Du et al. (2011) allocates berths considering fuel consumption and has a good review on other berth allocation literature. Well-established OR departments at many airlines have addressed the severe economical impact of flight delays and how to mitigate the effects of delays through disruption management based on OR. In 2008 the Joint Economic Committee under the U.S. Congress published a report estimating the infused cost to the American society to more than \$40 billion (JEC, 2008). The order of magnitude of the cost of disruptions has later been confirmed in a more theoretically profound study by Ball et al. (2010) even though their final estimate is $\approx 20\%$ lower. Both Rakshit et al. (1996) and Yu et al. (2003) document significant savings by implementing real-time decision support systems to handle the disruptions at major US airlines where the later estimates the annual saving to amount to \$40 million for Continental Airlines.

Disruption management research for airlines generally deals with recovering the 3 resource areas aircraft, crew and passengers. The full problem of optimizing all of these areas simultaneously is, however, so complex that no work has been published so far, which cover all 3 areas in one single integrated model. Most of the published models address one single resource. A few of the models focus on one resource area, while including specific aspects of other areas. For a good general introduction to disruption management in the airline industry the reader is referred to Yu and Qi (2004) and Barnhart (2009). The paper of Kohl et al. (2007) describes a large scale EU-funded project, called Descartes, which addresses various aspects of disruption management for all 3 resource areas. The reader is also referred to an extensive survey of operations research used for disruption management in the airline industry by Clausen et al. (2009). In order to adapt disruption management techniques applied to the airline industry to the liner shipping industry the aircraft recovery problem resembles vessel recovery and the recovery of passenger itineraries resembles container recovery. Since liner shipping companies do not have to deal with crew recovery, this literature review will only focus on aircraft and passenger recovery.

The first model on the *Aircraft Schedule Recovery Problem*, presented in the literature, is a network flow model by Teodorović and Guberinić (1984), who contributed by solving small problems with 3 aircraft and 8 flights. This work was extended by Teodorović and Stojković who extended the model in later papers. The solvable problem sizes still remained small with 14 aircraft and 80 flights. Jarrah et al. (1993) presented the first work, which were applicable in practice based on instances from United Airlines. They published 2 models, which in combination were capable of producing solutions handling all 3 traditional recovery techniques delays, swaps and cancellations. The drawback of handling this in 2 separate models was that delays and cancellations could not be traded off against each other. This drawback was resolved in the work by Yan and Yang (1996) who were capable of trading off delays, swaps and cancellations in one single model based on a time-line network. Thengvall et al. (2001) extended this model to also include so-called protection arcs, which serve the purpose of keeping the proposed solutions somewhat similar to the original schedule. This is important for real-life application of the suggested solutions as an unlimited number of changes cannot be applied to the schedule last minute. The work by Dienst et al. (2012) extends this model to also cover aircraft specific maintenances and preferences in an aircraft specific recovery model.

The *Passenger Recovery Problem* is an area of disruption management, which has been addressed to a rather limited extent by published research. Our observation from airlines show that most of these use a sequential re-accommodation process, which is carried out after an aircraft recovery

schedule has been decided upon. Vaaben and Alves (2009) do a comparison of sequential passenger re-accommodation with re-accommodation based on an MIP-model. The main contribution in the area of passenger recovery is done by Bratu and Barnhart (2006), who present two models. Both are basically aircraft recovery models with some crew recovery guidance. One of them also includes passenger recovery, but is not solvable in real time. The other one is solvable but does not include complete passenger recovery. Instead it penalizes passenger miss-connections.

The work by Marla et al. (2011) extends on the work by Dienst et al. (2012) and Bratu and Barnhart (2006) by doing aircraft specific recovery with penalized passenger miss-connections, while at the same time also introducing the additional recovery technique of *speed changes*, which enables the model to balance the trade-off between passenger delay cost and fuel burn cost in a network perspective. The purpose of the present paper is to investigate if the application of similar disruption recovery techniques in a liner shipping context will be beneficial.

4.4 The Vessel Schedule Recovery Problem - (VSRP)

A given disruption scenario consists of a set of vessels V , a set of ports P , and a time horizon consisting of discrete time-slots $t \in T$. The time slots are discretized on port basis as terminal crews handling the cargo operate in shifts, which are paid for in full, even if arriving in the middle of a shift. Hence we only allow vessels arriving at the beginning of shifts. Reducing the graph to time-slots based on these shifts, also has the advantage of reducing the graph size, although this is a minor simplification of the problem. For each vessel $v \in V$, the current location and a planned schedule consisting of an ordered set of port calls $H_v \subseteq P$ are known within the recovery horizon, a port call A can precede a port call B , $A < B$ in H_v . A set of possible sailings, i.e. directed edges, L_h are said to *cover* a port call $h \in H_v$. Each L_h represent a sailing with a different speed.

The recovery horizon, T , is an input to the model given by the user, based on the disruption in question. Inter continental services will often recover by speeding during ocean crossing, making the arrival at first port after an ocean crossing a good horizon, severe disruptions might require two ocean crossings. Feeders recovering at arrival to their hub port call would save many missed transshipments giving an obvious horizon. In combination with a limited geographical dimension this ensures that the disruption does not spread to the entire network.

The disruption scenario includes a set of container groups C with planned transportation scenarios on the schedules of V . A feasible solution to an instance of the VSRP is to find a sailing for each $v \in V$ starting at the current position of v and ending on the planned schedule no later than the time of the recovery horizon. The solution must respect the minimum and maximum speed of the vessel and the constraints defined regarding ports allowed for omission or port call swaps. The optimal solution is the feasible solution of minimum cost, when considering the cost of sailing in terms of bunker and port fees along with a strategic penalty on container groups not delivered “on-time” or misconnecting altogether.

4.4.1 Graph topology

A disruption scenario is conceptualized as a directed graph in a time-space network similar to the one used by Thengvall et al. (2000, 2001, 2003), Marla et al. (2011) and Dienst et al. (2012). The horizontal axis corresponds to a point in time within the given planning horizon, and the vertical axis corresponds to a geographical position; a port in the context of VSRP. A simple example of a time-space network is presented in Figure 4.3(a). Here, two geographical positions are given and a vessel can connect from the initial position A to the next position B with three different speeds.

A directed graph $G = (N, E)$ with node set $N = \{p^t \in N | p \in P, t \in T\}$ where p^t denotes port p at time t representing the time-space network. n^- and n^+ denotes the in- and out-going edges of node $n \in N$ respectively. $N_v \subseteq N$ is the set of all nodes for vessel $v \in V$. The set consists of a source node n_s^v corresponding to the current position of the vessel and a sink node n_t^v corresponding to the scheduled position at the end of the recovery horizon. Additional nodes are created for the set of port calls $h \in H_v$ within a time window of $\{a_v^h, b_v^h\}$ defining the earliest

and latest arrival time respectively given the vessels minimum and maximum speed, the current position and the remaining set of port calls.

Define the edge set $E = E_s \cup E_g$ where E_s represents a sailing of a vessel $v \in V$ such that $E_s = \{(p^t, q^{t'}) | p^t, q^{t'} \in N, p \neq q, t \leq t'\}$ and $E_g = \{(p^t, p^{t'}) | p^t, p^{t'} \in N, t < t'\}$. The duration of a port call is fixed for each vessel $v \in V$ according to the scheduled port call duration from the original schedule. Because the port call duration is fixed port call edges E_g are included in the sailing edges E_s , thereby removing the set E_g as seen in Figure 4.3(b). Including the edge set E_g in E_s reduces the number of columns in the mathematical model. For illustrative purposes the port call edges are still visualized in Figures 4.3(c) and 4.3(d), while the remainder of the figures in this paper only visualize the combined edges.

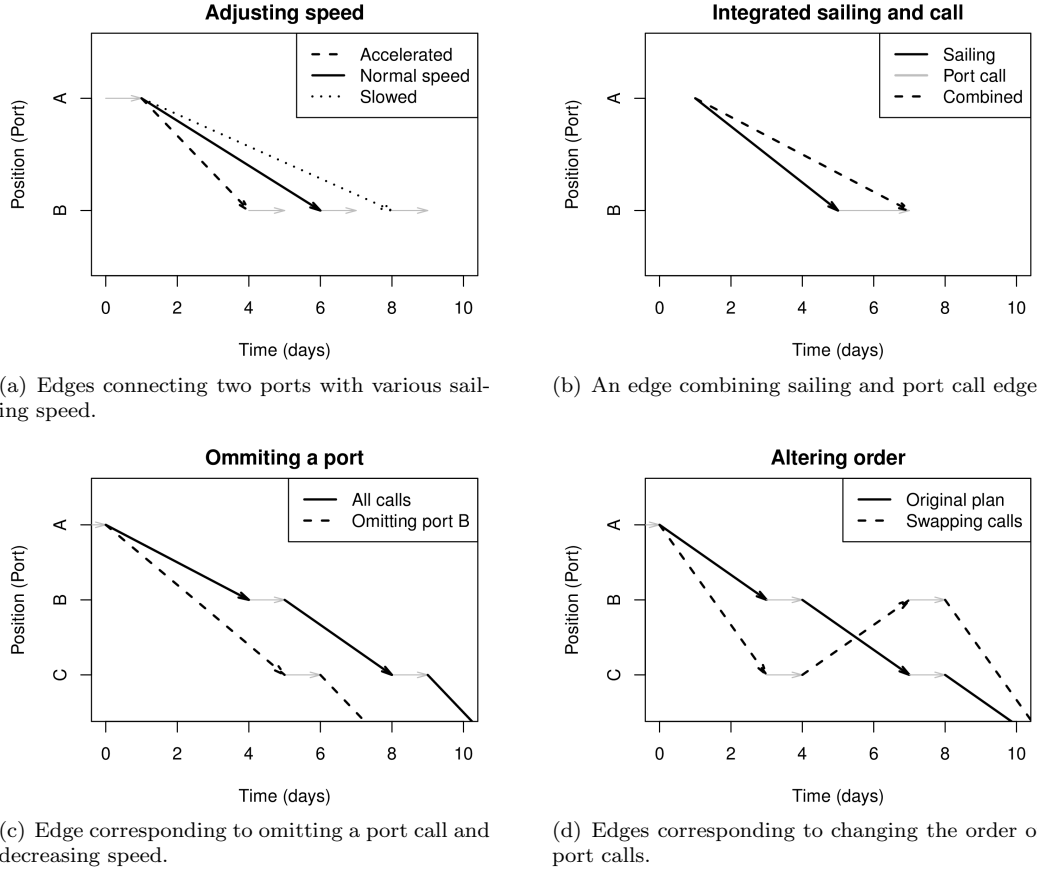


Figure 4.3: Possible moves in the time-space network model. Port call edges are gray.

The edge sets $E_v \subseteq E_s$ are the edges that define feasible sailings among the nodes of N_v for a given vessel $v \in V$. $c_e^v \in \mathbb{R}_+$ is the cost of using edge $e \in E_v$ for vessel type $v \in V$ consisting of the bunker cost at a given speed and port fee for port $p = \text{target}(e)$. t_e^v is the time it takes to traverse edge $e \in E_s$ given speed, distance and port call time. The edge set $E_s = \bigcup_{v \in V} E_v$ is defined according to the planned schedule and the possible recovery actions defined below:

- **Adjusting vessel speed** (Figure 4.3(a))
In the span of the minimum and maximum speed of vessel $v \in V$ several edges may connect ports A and B. Define the set of edges $L_h \subset E_v$ covering port call $h \in H_v$ as $L_h = \{(A^t, B^{t'}) | A, B \in H_v, A < B, t \leq b_v^A, a_v^B \leq t' \leq b_v^B, t < t', \forall t' = a_v^B + K \cdot \delta_B\}$ where K is a positive integer denoting the shift and δ_B is the duration of a shift at terminal B.
- **Omitting a port call** (Figure 4.3(c))

Vessels might omit port calls to recover a delay or simply to save the port cost. Omitting port calls will result in miss-connected containers. Allowing to omit port B on a sailing from port A via port B to port C corresponds to having an edge (A^{t_A}, C^{t_C}) where $t_C - t_A$ corresponds to the sailing time. Edges L_h with differing sail speeds must be created as described in above bullet.

- **Swap order of calls** (Figure 4.3(d))

In some cases, a delayed vessel needs to call a number of ports close to each other. It might be possible to swap port calls within a designated geographical area. In the time-space network a swap is included by adding, first an omitting edge, followed by an edge back to the original port call. Again this must be executed for differing vessels speeds, as described in first bullet.

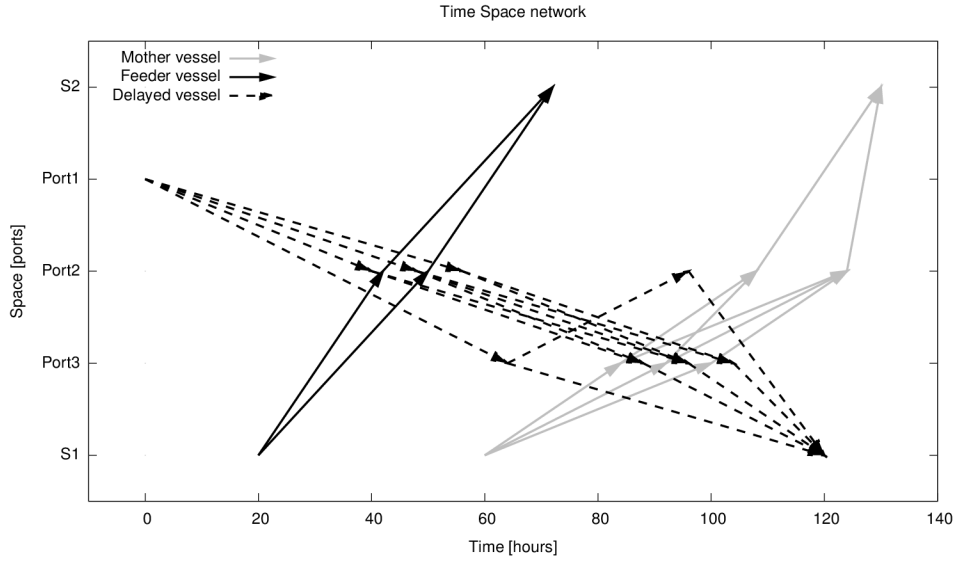


Figure 4.4: Example of a time-space network for a test problem with three vessels, sinks and sources, three ports, speed adjusting edges, and port swap for the *delayed* vessel. In the network only the edges taking part in a feasible path are shown.

Figure 4.4 gives an example of a full time-space network for a small test instance. Three vessels are affected by the delay of the *delayed* vessel.

The set of vessels is

$$V = \{\text{delayed}, \text{feeder}, \text{mother}\} = \{d, f, m\}$$

and for each vessel a set of port calls is given. These are

$$H_d = \{P_1, P_2, P_3, S_1\}$$

$$H_f = \{S_1, P_2, S_2\}$$

$$H_m = \{S_1, P_3, P_2, S_2\}$$

where P_i is Port i and S_i corresponds to onward sailing according to schedule. For each of the port calls $h \in H_v$ a set of possible sailings L_h covering the call is given. As an example vessel d has the set of four possible sailings/legs covering the call in Port 2:

$$L_{(d, P_2)} = \left\{ \begin{array}{ll} (P_1, 0) \rightarrow (P_2, 38) & , \quad (P_1, 0) \rightarrow (P_2, 48) \\ (P_1, 0) \rightarrow (P_2, 58) & , \quad (P_3, 62) \rightarrow (P_2, 98) \end{array} \right\} .$$

The cost of each of these edges is the sum of the bunker cost from sailing with the necessary speed between the ports and the cost of calling Port 2. The cost of using leg $(P_1, 0) \rightarrow (P_2, 38)$ is higher than the cost of using leg $(P_1, 0) \rightarrow (P_2, 58)$ as the sailing time is smaller ($38 < 58$) resulting in a higher sailing speed and consequently an increased bunker fuel burn.

The problem has characteristics that are not directly reflected in the graph. These are the flow of containers, extended port stays due to omissions, limits on the capacity of a port, and port closure in a period of time. The extended port stay due to an omission can readily be handled in the graph construction by adjusting the duration of the set of sailing edges in E_s , that represent the omission. This has not been done to simplify modeling, as the effect will small. The port capacity issue can be modeled by constraining the number of vessels arriving (or used legs) at each port in each given time interval. Port closures are included by removing all edges corresponding to arriving at a port while it is closed.

4.4.2 Transportation scenarios - the impact of a recovery on the affected cargo

In order to evaluate which container groups will suffer from missed onward connections and delays we define a transportation scenario for each container group in terms of their origin, destination and planned transshipment points. $B_c \in H_v$ is defined as the origin port for a container group $c \in C$ and the port call where vessel v picks up the container group. Similarly, we define $T_c \in H_w$ as the destination port for container group $c \in C$ and the port call where vessel w delivers the container group. Intermediate planned transshipment points for each container group $c \in C$ are defined by the ordered set $I_c = (I_c^1, \dots, I_c^m)$. Here $I_c^i = (h_v^i, h_w^i) \in (H_v, H_w)$ is a pair of calls for different vessels ($v, w \in V | v \neq w$) constituting a transshipment. Each container group c has m^c transshipments. M_c^e is the set of all non-connecting edges of $e \in L_h$ that result in miss-connection of container group $c \in C$. $c_c^d \in \mathbb{R}_+$ is the cost of a delay to container group $c \in C$ exceeding a day of the planned arrival and $c_c^m \in \mathbb{R}_+$ is the cost of one or several misconnections to container group $c \in C$, which is added to the delay penalty in the model.

The cost of delaying the arrival of a container at its destination is to a large extent related to the loss of goodwill from the affected customers. This may vary by the type of container and the importance of the customer to the liner shipping company. In general refrigerated containers are more costly to delay than non-refrigerated, but more detailed classification by container type and customer value may be applied. The cost classifications used in the case-studies in this paper have been supplied by Maersk Line and are based on their internal approximations of these costs.

4.4.3 Mathematical model

The mathematical model is inspired by the work within aircraft recovery with speed-changes by Marla et al. (2011). Like others before Marla et al. (e.g. Dienst et al. (2012)) we use a time space graph as the underlying network, but reformulate the model to address the set of available recovery techniques, which are applicable to the VSRP.

Define binary variables (x_e) for each edge $e \in E_s$ set to 1 iff the edge is sailed in the solution. Define binary variables (z_h) for each port call $h \in H_v \quad \forall v \in V$ set to 1 iff call h is omitted. For each container group c we define binary variables $o_c \in \{0, 1\}$ indicating whether the container group is delayed or not and y_c to account for container groups misconnecting. $O_e^c \in \{0, 1\}$ is a constant set to 1 iff container group $c \in C$ is delayed when arriving by edge $e \in L_{T_c}$. $M_c \in \mathbb{Z}_+$ is an upper bound on the number of transshipments for container group $c \in C$.

$$S_v^n = \begin{cases} -1 & , n = n_s^v \\ 1 & , n = n_t^v \\ 0 & \text{Otherwise} \end{cases}$$

is applied to the flow conservation constraints.

Minimize:

$$\sum_{v \in V} \sum_{h \in H_v} \sum_{e \in L_h} c_e^v x_e + \sum_{c \in C} [c_c^m y_c + c_c^d o_c] \quad (4.1)$$

Subject To:

$$\sum_{e \in L_h} x_e + z_h = 1 \quad \forall v \in V, h \in H_v \quad (4.2)$$

$$\sum_{e \in n^-} x_e - \sum_{e \in n^+} x_e = S_v^n \quad \forall v \in V, n \in N_v \quad (4.3)$$

$$y_c \leq o_c \quad \forall c \in C \quad (4.4)$$

$$\sum_{e \in L_{T_c}} O_e^c x_e \leq o_c \quad \forall c \in C \quad (4.5)$$

$$z_h \leq y_c \quad \forall c \in C, \forall h \in B_c \cup I_c \cup T_c \quad (4.6)$$

$$x_e + \sum_{\lambda \in M_c^e} x_\lambda \leq 1 + y_c \quad \forall c \in C, e \in \{L_h | h \in B_c \cup I_c \cup T_c\} \quad (4.7)$$

$$x_e \in \{0, 1\} \quad \forall e \in E_s \quad (4.8)$$

$$z_h \in \mathbb{R}_+ \quad \forall v \in V, h \in H_v \quad (4.9)$$

$$y_c, o_c \in \mathbb{R}_+ \quad \forall c \in C \quad (4.10)$$

The objective function (4.1) minimizes the cost of operating vessels at the given speeds, the port calls performed along with the penalties incurred from delaying or misconnecting cargo. The weighted sum scalarization (Ehrgott, 2005), the ϵ -constraint method (Ehrgott, 2005), and variable fixing has been implemented for the VSRP with promising results in the thesis by Dirksen (2011).

Constraints (4.2) are *Set-Partitioning* constraints ensuring that each scheduled port call for each vessel is either called by some sailing or omitted. (4.3) are *Flow-Conservation* constraints. Combined with the binary domain of variables x_e and z_h they define feasible vessel flows through the time-space network. A misconnection is by definition also a delay of a container group and hence the misconnection penalty is added to the delay penalty. This is expressed in (4.4).

Each container group has a planned arrival time upon which it can be decided whether or not a given sailing to the destination will cause the containers to be delayed. Constraints (4.5) ensure that o_c takes the value 1 iff container group c is delayed when arriving via the sailing represented by edge $e \in E_s$. The right hand side does not have to be multiplied despite the number of summed variables may be larger than one due to the cover constraint (4.2) as this constraint ensures that only one incoming edge $x_e, e \in L_{T_c}$ can have flow. Constraints (4.6) ensure that if a port call is omitted, which had a planned (un)load of container group $c \in C$, the container group is misconnected. Constraints (4.7) are coherence constraints ensuring the detection of container groups' miss-connections due to late arrivals in transshipment ports. For each of the possible inbound sailings of a container transshipment a constraint is generated. On the left-hand side the decision variable corresponding to a given sailing, x_e , is added to the sum of all decision variables corresponding to having onward sailing resulting in miss-connections, $\lambda \in M_c^e$.

The constraint is illustrated in Figure 4.5. When implementing the constraint the variables corresponding to inbound sailings are summed.

The variable x_e is required to be binary, whereas the remaining variables are only required to be non-negative. Binary x_e combined with constraints (4.2) implies z_h to be binary. Given the binary domains of x_e and z_h combined with constraints (4.6), (4.7) and a minimization implies y_c to be binary. Finally, Minimization, binary domains of x_e and y_c combined with constraints (4.4) and (4.5) imply that o_c is binary.

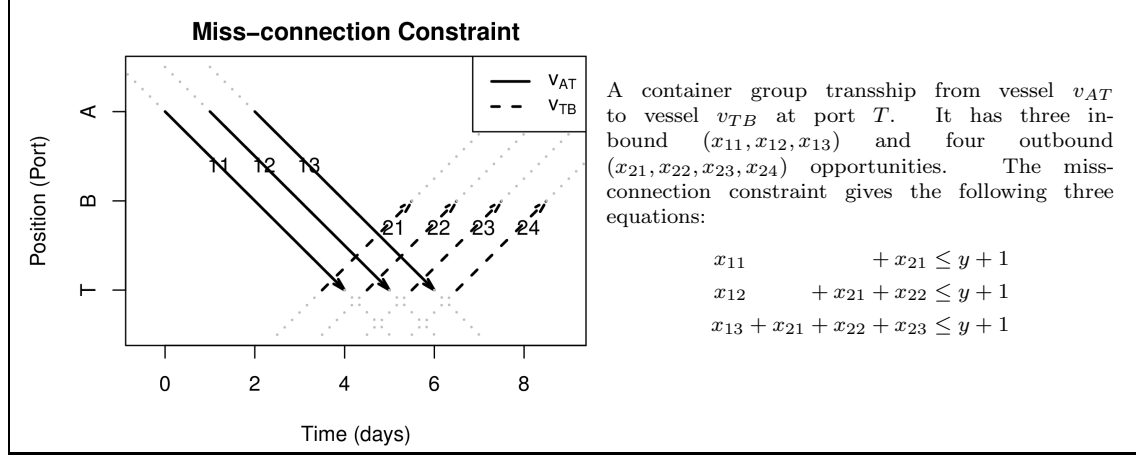


Figure 4.5: Example of the miss-connection constraint (4.7).

4.4.4 Model extensions

The model can be extended to incorporate additional features of a given problem instance such as the berth occupation constraint.

$$\sum_{v \in V} \sum_{h \in H_v} \sum_{e \in L_h} U_e^{pt} x_e \leq 1 \quad \forall p \in P, t \in T \quad (4.11)$$

$U_e^{pt} \in \{0, 1\}$ is a constant set to 1 iff edge $e \in L_h$ occupy a berth in port $p \in P$ in time slot $t \in T$. The constraint ensures that only a single vessel can enter and use a berth at a given time. This constraint will not handle berth allocation in general, which specified methods exist for, as mentioned in literature review. But when several vessels have to compete for a single berth available at a terminal, this constraint can be used to model the liner shipping company's choice of prioritization, irrespective of the terminal's options.

4.4.5 Complexity

The VSRP is NP-hard if omissions of ports is allowed, or if port swaps are allowed. Even if only one of the recovery actions is allowed, the problem is NP-hard as shown in the following: If omissions of ports are allowed in VSRP, the NP-hardness can be proved by reduction from the *0-1 Knapsack Problem (KP)*. Given an instance of the KP with a knapsack of capacity c , and n items having profit p_i and weight w_i , we transform it to an instance of the VSRP by using a single vessel and n ports which can be omitted. The cost of omitting a port is set to $-p_i$ and the duration of a port call is set to w_i . Sail times between ports are set to zero, and the recovery horizon is set to c , ensuring that a maximum profit subset of the items is chosen satisfying the capacity of the knapsack.

If port swaps are allowed in VSRP, the NP-hardness is shown by reduction from the *Traveling Salesman Problem (TSP)*. Given an instance of TSP with n nodes and edge costs c_{ij} , we construct an instance of the VSRP by introducing n ports which can be visited in arbitrary order. Port calls and travel times are set to zero, while the sail cost between ports is c_{ij} . The cost of omitting a port is set to infinity ensuring that all ports are visited following the shortest Hamiltonian cycle.

The above reductions prove that the VSRP with allowed omissions is weakly \mathcal{NP} -hard and the VSRP with multiple omissions to be strongly \mathcal{NP} -hard. Extended proofs for the \mathcal{NP} -completeness of the VSRP may be found in Dirksen (2011).

4.5 Computational results

The program has been run on a MacBook Pro with 2.26 GHz processor and 2 GB of memory running Mac OSX using IBM ILOG CPLEX 12.2.0.0 as MIP solver. To test the performance and applicability of the developed model, it has been run on four real instances and a number of auto generated instances.

4.5.1 Real-life Cases from Maersk Line

The cases used to evaluate the VSRP are based on historical events at Maersk Line (ML). They are selected to represent the most common disruption scenarios and recovery options. Each case includes information about vessel schedules, port distances, container movements, recovery options, vessel speeds, and costs. ML handles these types of disruptions on a daily basis. The purpose of the cases is to test the suggested model, but also to clarify typical disruptions and how they are currently handled. An overview of the cases is given followed by a detailed presentation. The cases are

1. **A Delayed Vessel**

The vessel Maersk Sarnia is delayed out of Asia due to bad weather. The vessel is, filled with cargo, about to cross the Pacific Ocean and unload in Mexico and Panama.

2. **A Port Closure**

The port Le Havre in France is closed due to a strike. The vessel Maersk Eindhoven arriving with cargo from Asia can either wait for the port to open (giving an expected 48 hour delay) or omit the call in Le Havre.

3. **A Berth Prioritization**

The port in Jawaharlal Nehru (India) does not have the capacity for a ME3-service vessel and a MECL1-service vessel to port at the same time. As the MECL1-service vessel is delayed and the vessels will arrive at the port simultaneously, it is necessary to decide which vessel to handle first.

4. **Expected Congestion**

The feeder vessel Maersk Ravenna is planned to call three Colombian ports. Due to port maintenance at the last port to call, a delaying congestion is expected if arriving as planned. ML has to decide if the plan should be changed to avoid the congestion.

4.5.2 Case results

The computational results for the cases are promising. Good recovery strategies have been generated within 5 seconds, which proves the model applicable as a real-time decision support tool for liner shipping companies. The optimization based recovery strategies are generated with a strategic penalty for delaying and misconnecting containers. The two penalties are given the same value, i.e. $c_c^m = c_c^d$. For each of the cases discretization of the time horizon is $\delta = 3$ hours. Table 4.1 shows different size measures for the four cases. The results from the optimized runs (**OPT**) have been compared to the real life solution (**RS**). **RS** is the realized sailings for the affected vessels and the realized impact on containers. All presented costs are relative to the real cost to preserve the relativeness of bunker, port fees and container impact of a solution. However, the costs have no relation to real life costs.

An overview is given in Table 4.2. The results clearly show potential in the mathematical model. The experts at ML have indicated that in two out of the four cases they would prefer **OPT**, in one **OPT** is the same as **RS**, and in the last **RS** is preferred. However, in the case where **RS** is preferred, the recovery strategy is based on re-flowing cargo, which is not considered by the VSRP. The tendency is clearly that the model generates competitive solutions and would be a substantial support to the operator resulting in better recovery solutions using significantly less time. However, based on just four cases it is not reasonable to conclude that the optimized

solutions are generally superior. The computational times are less than 5 seconds with CPLEX consuming roughly half the execution time, while graph generation consumes the rest. Please note that Case 1 (A Delayed Vessel) has a much longer planning horizon than the remaining cases, which accounts for the increase in running times. Even for Case 1 the solution time is indeed acceptable for a operational application.

4.5.3 Case 1 (A Delayed Vessel)

Within the planning horizon of Case 1 Maersk Sarnia delivers containers to a single ML vessel in Lazaro Cardenas and seven ML vessels in Balboa. Each vessel may be delayed to the originally planned arrival time. The vessel Maersk Sarnia is allowed to omit Yokohama and either Lazaro Cardenas or Balboa. The **OPT** is structurally different to **RS**. Both are plotted in Figure 4.6. ML has chosen to call all ports with a speed increase (**RS**). However, the speed-up is not sufficient to reach the head haul ports in time. The optimized solutions (**OPT**) is to omit the call in Yokohama resulting in 400 misconnected containers while the remaining ports are called in time. The combined costs and penalties of **RS** are 24% higher than the costs and penalties of **OPT**. The experts at ML confirm that omitting Yokohama was a superior solution and note, that they were unable to convince a single important stakeholder of the superiority of this solution. It is very clear that the generalized mathematical assessment provided by a decision support tool would have been a strong argument in the discussion.

4.5.4 Case 2 (A Port Closure)

In Case 2 (A Port Closure) either Le Havre is called 48 hours delayed, or Rotterdam is called at the planned time. In Le Havre 649 containers need to be loaded and 1911 need to be unloaded. The time-space network of the case is presented in Figure 4.7.

Again **OPT** is different in structure compared to **RS** (Figure 4.8). However, as noted earlier **RS** is based on re-flowing containers not considered by the VSRP. Surprisingly, the data for the suggested solutions show that **OPT** is a better alternative with respect to cost. In real life the delay turned out to be 72 hours and a solution was obtained by allowing to merge two port calls. This option was not available to the model and hence the results are not comparable.

4.5.5 Case 3 (A Berth Prioritization)

In the third case, the additional berth occupation constraint (4.11) is added to ensure that the vessels call the port in India one at a time. The berth prioritization case is interesting as four of the connecting ML vessels may be delayed significantly and still reach their next port to call. **OPT** and **RS** result in the same solution presented in Figure 4.9. The runs confirm the decision of **RS** and verify the applicability of a decision support system in an operational setting, providing fast solutions. In this case the decision would have been reached in a matter of seconds as opposed to hours.

Case	V	PC	CG	C	RH	N	E	x_e	z_h	y_c / o_c	Constraints
1	8	26	23	5145	961	301	7073	7073	10	23	1706
2	6	22	19	12358	969	118	290	290	10	19	122
3	10	33	24	5671	548	171	411	411	13	24	221
4	1	5	6	838	166	103	416	416	3	6	300

Table 4.1: An overview of the relative sizes of the cases in terms of the number of vessels (**V**), the number of port calls in the scenario (**PC**), the number of container groups included (**CG**), the total number of containers (**C**), the recovery horizon in hours (**RH**), the size of the graph (N, E), and the number of variables (x_e, z_h, y_c, o_c).

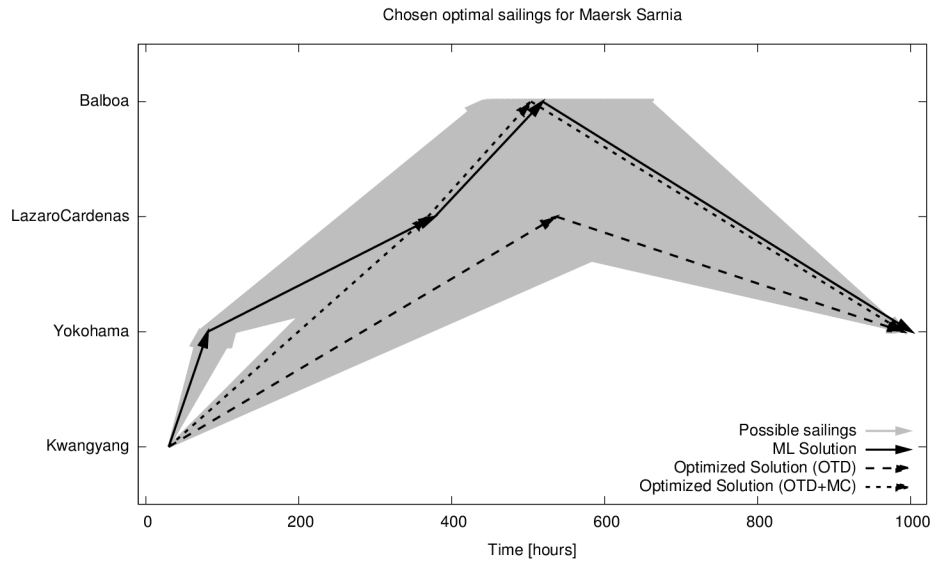


Figure 4.6: Case 1: Suggested recovery solutions for Case 1 (A Delayed Vessel).

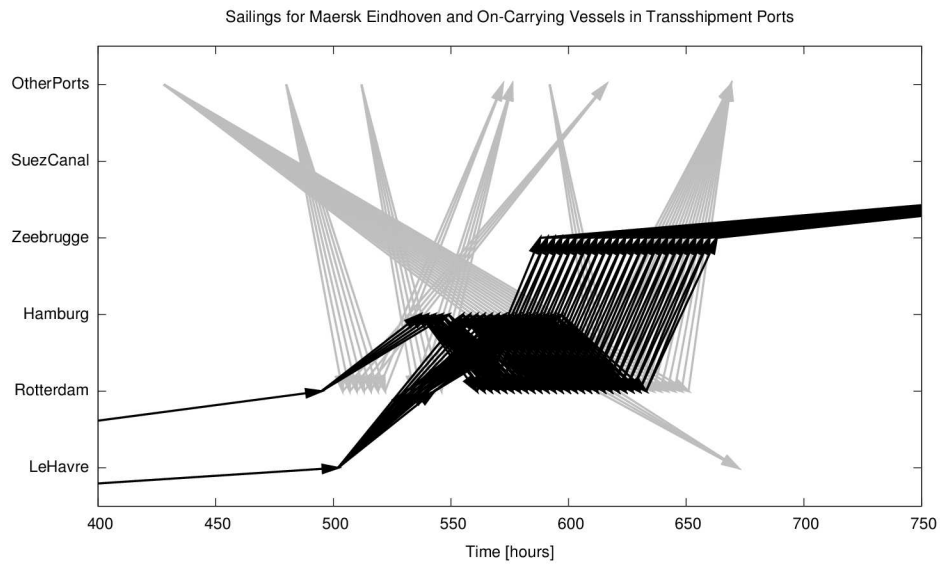


Figure 4.7: Time-space network for Case 2.

		Sailing Cost		Delays		Misconnections		Solve	Best
		RS	OPT	RS	OPT	RS	OPT	Time	
Case	1	1,000,000	914,063	(2449)	(0)	(26)	(400)	4.529	OPT
	2	1,000,000	977,392	(3111)	(3111)	(58)	(58)	0.718	RS
	3	1,000,000	1,000,000	(687)	(687)	(0)	(0)	0.681	Equal
	4	1,000,000	1,033,334	(222)	(0)	(0)	(0)	0.518	OPT

Table 4.2: Overview of results for the cases. The costs are relative, the container impact in units, and the time to solve in seconds. The best-column shows which solution the ML experts would prefer today.

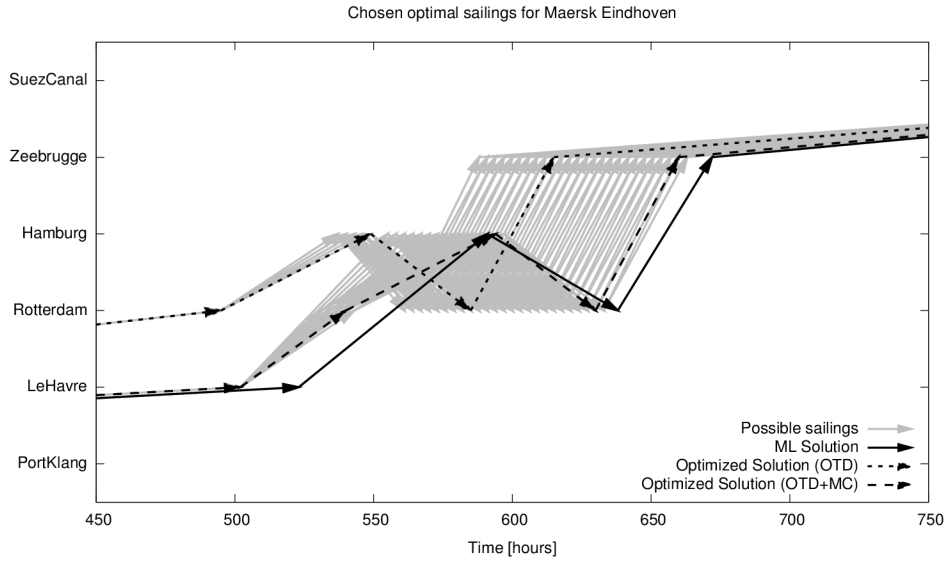


Figure 4.8: Suggested recovery solutions for Case 2.

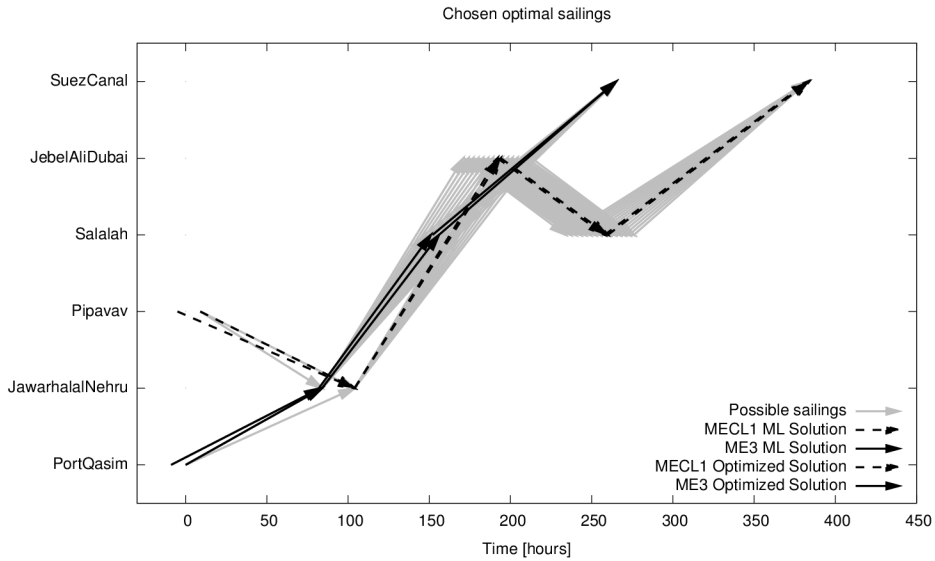


Figure 4.9: Time-space network and solution for Case 3. The ME3-vessel (full line) or the MECL1-vessel (dashed line) calls Jawaharlal Nehru in India first.

4.5.6 Case 4 (Expected Congestion)

The last case where a feeder vessel is expecting port congestions in the last port differs completely from the former cases. The feeder only carries direct import and export cargo to and from Colombia, meaning that no additional vessels need to be taken into account and that a single run is generated as misconnections are not possible. The expected port delay (of 24 hours if Santa Marta is called after $t = 100$) combined with the possibility of calling the three ports in Colombia in any order defines the problem. The time-space network of possible sailings along with the solutions is given in Figure 4.10. **RS** was to alter the order of the port calls to ensure that Santa Marta was visited long before the expected congestion. This resulted in a delay to the cargo in Cartagena. Contrary to **RS**, **OPT** suggests continuing as planned, but speeding up to

arrive at Santa Marta before the expected congestion. This solution displays slightly increased bunker cost but ensures that all containers are delivered on time. According to the experts at ML, the optimized solutions should have been implemented. The costs and penalties reveal a saving amounting to a stunning 58%.

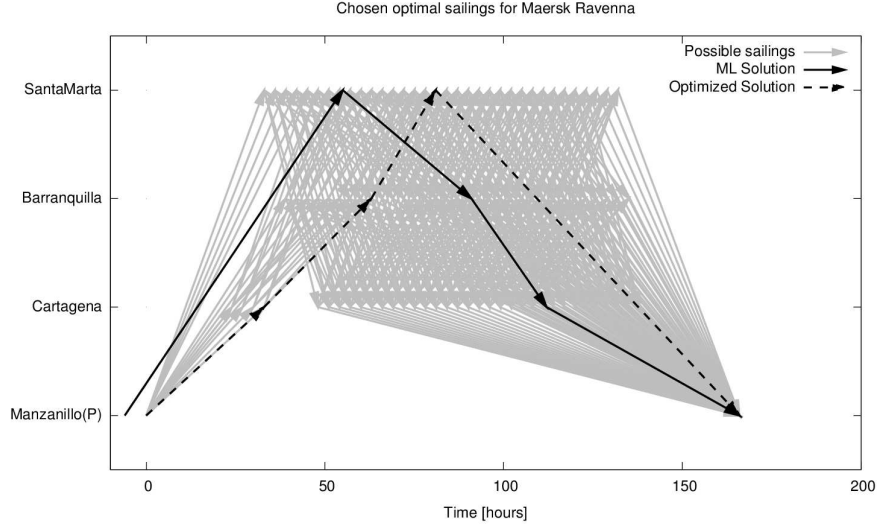


Figure 4.10: Suggested recovery solutions for Case 4 (Expected Congestion) in the time-space network.

4.5.7 Auto generated test instances

The four cases utilize different parts of the solution space satisfactorily, but lack in size and are thus relatively fast to solve. To test the scalability of the model, a set of random instances have been generated, refer to Figure 4.11 for an example. η^2 ports are placed in a squared grid, where distances and sailing times are proportional to Euclidean distances. Vessels are generated with a random schedule of $\kappa < \eta^2$ ports to call. Container itineraries are generated such that each intermediate call for each vessel and arriving container group is added with some probability. For these instances the computational time grow with increasing number of calls per vessel and number of vessels in instance as seen in Figure 4.12. It can be seen that the computational time handles an increased number of vessels well, but is impacted harder by an increased number of port calls. It seems viable that the model will solve in minutes for instances with up to 10 vessels and port calls making it viable for use in a wide range of real world problems. For more details on how the instances are generated and details on computational time please refer to the thesis by Dirksen Dirksen (2011).

4.6 Conclusion and future work

To the best of our knowledge this paper is the first literature on decision support for disruption management in a liner shipping network. We have presented a novel mathematical model for the Vessel Schedule Recovery Problem (VSRP). The model addresses frequently occurring disruption scenarios in the liner shipping industry. The model is based on disruption management work from airline industry and adapted to liner shipping. We show the VSRP to be NP-complete. The model is solved using a MIP solver and computational experiments indicate that the model can be solved within ten seconds for instances corresponding to a standard disruption scenario in a global liner shipping network. Computational results for four real-life cases show similar or improved solutions to historic data. The solutions have been verified by experienced planners. A set of generic test instances have been provided and computational results indicate that the model

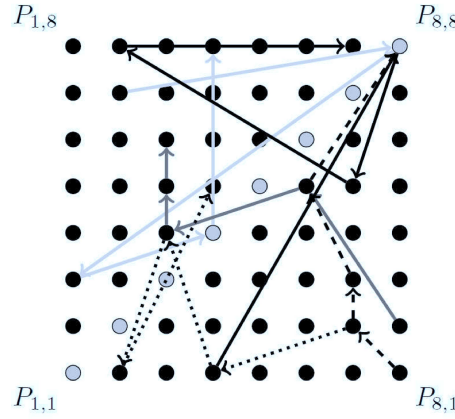


Figure 4.11: Graphical explanation of the standard way random instances of the VSRP are generated.

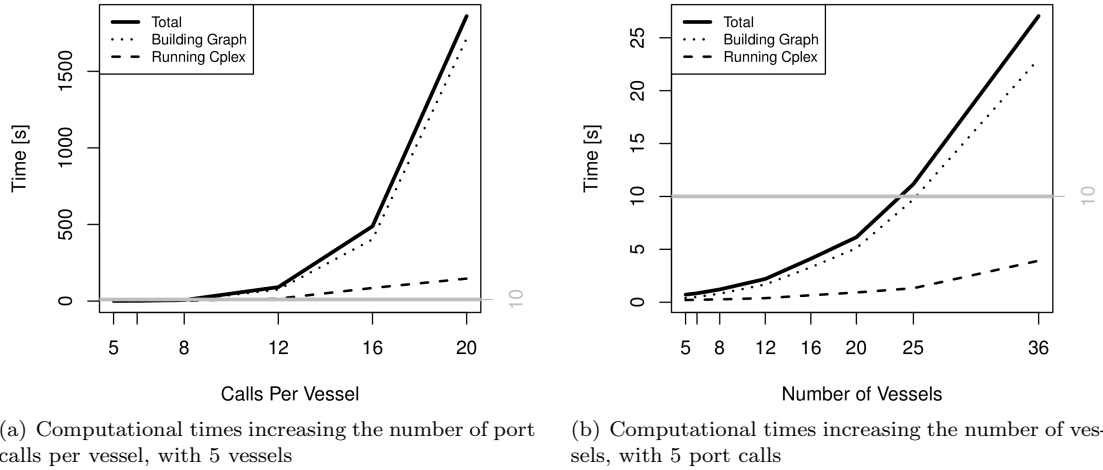


Figure 4.12: Computational times for generic generated problems with varying number of ports and vessels respectively. The times are average values based on 5 repeated runs.

is capable of handling larger disruption scenarios than the real-life cases in seconds. However, with an increasing number of vessels, the computational time show exponential growth and can no longer reach an optimal solution within ten seconds, for larger instances. An analysis of the four real life cases, show that a disruption allowing to omit a port call or swap port calls may ensure timely delivery of cargo without having to increase speed and hence, a decision support tool based on the VSRP may aid in decreasing the number of delays in a liner shipping network, while maintaining a *slow steaming* policy. This initial work on disruption management in liner shipping show potential for interesting extensions. Other recovery modes than the three considered (speed adjustment, port call omission and port call swap) could be investigated, e.g. reducing the time spent at port by unloading but not loading, merging port calls or adding protection arcs. Another extension would be to reroute the non-satisfied cargo on the remaining, or even third party network. The connection with berth scheduling problems with disruption of fixed scheduled services as considered here could also be explored further.

Acknowledgements

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Chapter 5

A matheuristic for the Liner Shipping Network Design Problem

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Abstract We present a *matheuristic*, an integer programming based heuristic, for the Liner Shipping Network Design Problem. The liner Shipping Network Design Problem is to find a set of container shipping routes defining a capacitated network on which a set of demands can be transported. The cargo transports make extensive use of transshipments between services. The services have weekly frequency. The weekly frequency is an industry standard as liner shipping companies publish a set of scheduled services to their customers. The heuristic applies a greedy construction heuristic, where the liner shipping network is split into routes solving a multiple quadratic knapsack problem. The construction heuristic is combined with an improvement heuristic with a neighborhood defined as a mixed integer program. The mixed integer program optimizes the removal and insertion of several port calls on a liner shipping service. The objective function is based on evaluation functions for revenue and transshipment of cargo along with in/decrease of vessel- and operational cost for the current solution. The evaluation functions may be used by heuristics in general to evaluate changes to a network design without solving the underlying large scale multicommodity flow problem. Computational results are reported for the benchmark suite *LINER-LIB 2012* and are the first results reported for a strict weekly frequency. The heuristic minimizes the operational cost and computational results are able to find profitable transportation networks for four out of six cases (12/18 instances). The heuristic shows overall good performance and is able to find profitable solutions within a very competitive execution time. The matheuristic is also evaluated as a decision support tool, where the initial solution is an existing network and optimization is allowed for a subset of the services considering the flow in the entire network. Results are promising for this approach.

Keywords: liner shipping, matheuristic, mathematical programming, network design

ROUTES	SERVICES
West Coast of North America - Asia	68
East Coast of North America - Asia	22
North America - Northern Europe	17
North America - Mediterranean	20
Asia - North Europe	35
Asia - Mediterranean	40
North America - East Coast of South America	8
North America - West Coast of South America	16
North America - North Coast of South America	26
Europe - East Coast of South America	10
Europe - West Coast of South America	8
Europe - North Coast of South America	10
Asia - East Coast of South America	8
Asia - West Coast of South America	13
South Africa - Europe	5
South Africa - North America	2
South Africa - Asia	19
West Africa - Europe	37
West Africa - North America	3
West Africa - Asia	18
Total	385

Table 5.1: Worldwide services from WSC (2011). Notes: Services may be counted on more than one route. "Asia" includes Australia and New Zealand. Source: ComPair Data World Liner Supply Report Summary, October 1, 2010; Drewry, Container Forecast Q3 2010; Drewry, Container Forecast Q4 2010.

5.1 Introduction

Liner shipping is the mass transit system of the ocean ways with regular scheduled services of varying capacity between geographical regions. A service is a sequence of port calls sailed by a number of vessels with a designated capacity. It is common for the industry to call every port on a service every week, which is achieved by the deployed vessels sailing exactly one week apart. Liner shipping and containerized transportation of goods over sea is a key component in today's supply chains. Approximately 400 liner shipping services are operated by a vessel fleet of close to 6000 container vessels (WSC (2011)). The services are distributed on regional areas as seen in Table 5.1. The liner shipping industry transports about 60% of the goods measured by value transported internationally by sea (WSC (2011)). The significance and magnitude of the liner shipping network makes the network design an important transportation problem. The network has high fixed asset costs in terms of the container vessels deployed and hence capacity utilization and network efficiency is crucial to a competitive liner shipping operation. At the same time maritime transport is accountable for an estimated 2.7% of the world's CO₂ emissions, whereof 25% is attributable to container ships alone (WSC, 2009). Fuel cost is the largest variable cost of operating a liner shipping network (Stopford, 2009). Performing optimization on the liner shipping network can have a huge impact on the trade of liner shipping as maximizing the revenue while considering variable operational cost may ensure a better capacity utilization in the network. Improved capacity utilization will increase profit for liner shipping companies, and give competitive freight rates for global goods. In due time optimization may target reducing the speed of the container fleet to decrease the CO₂ emissions from liner shipping in general as seen in the case of tramp shipping (Fagerholt et al., 2009).

The liner shipping network design problem (LSNDP) is to construct a set of non-simple cyclic services to form a capacitated network for the transport of containerized cargo. The network design maximises the revenue of container transport considering the cost of vessels deployed to services, overall fuel consumption, port call costs and cargo handling costs. Literature on the LSNDP is

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quite scarce (Brouer et al., 2012) compared to related maritime shipping transportation problems, but a surge in publications over recent years show increased interest in the LSNDP. The works of Agarwal and Ergun (2008); Alvarez (2009); Reinhardt and Pisinger (2011); Plum (2010); Brouer et al. (2012) reveal that the liner shipping network design problem is a very complex optimization problem, where current mathematical formulations and state-of-the-art exact solution methods cannot scale to realistic sized problem instances at the time of writing. One heuristic approach has been applied to large scale instances in (Alvarez, 2009; Brouer et al., 2012). A core concept in liner shipping is the transshipment of containers. More than 50% of cargoes are transported on more than one service from origin to destination. This means that we can model the LSNDP with an underlying multicommodity flow problem (MCF). Alvarez (2009) identifies the excessive time used for solving the MCF to evaluate a given network configuration as a bottleneck in local search methods. As a result, within reasonable computation time the tabu search by Brouer et al. (2012) only performs a limited search of the solution space of large scale instances.

In this paper, we present a matheuristic for solving the LSNDP. Matheuristics are an emerging field within optimization and are defined as methods exploiting the synergies of mathematical programming and metaheuristics (Maniezzo et al., 2009). The domain is wide and includes the use of mathematical programming techniques in a heuristic variant as well as deploying mathematical programming methods within a metaheuristic framework (Maniezzo et al., 2009). In the present paper we use mathematical programming to explore our neighborhood defined as the solution space of a mixed integer program designed to capture the complex interaction of the cargo allocation between routes.

One of the first approaches of using this technique was Franceschi et al. (2006) for the Distance-Constrained Capacitated Vehicle Routing Problem. The method has also been explored for the Split Delivery Vehicle Routing Problem by Chen et al. (2007), the Split Delivery Vehicle Routing Problem with minimum delivery amounts by Gulczynski et al. (2010), by Archetti et al. (2010) for the The Split Delivery Capacitated Team Orienteering Problem and lately in Gulczynski et al. (2011) for the Periodic Vehicle Routing Problem. In all cases the matheuristic solution method combining local search with an integer program as neighborhood has proven very successful compared to other state-of-the-art heuristics.

In the present paper we exploit this technique in a metaheuristic framework for the LSNDP. We make four main contributions: We present a construction heuristic for the LSNDP by transforming the problem into an instance of the multiple quadratic knapsack problem. Secondly, an improvement heuristic is applied to the solution of the construction heuristic. The improvement heuristic is a large neighborhood search defined as a mixed integer program inserting and removing port calls from a single service. Thirdly, the heuristic makes use of estimation functions for the change in a large MCF, in order to avoid the bottleneck of solving a large scale MCF. Once moves are applied to a service the neighborhood of subsequent services is based on an optimal solution of the MCF in order to decrease the error of the evaluation functions. The MCF is resolved using an advanced warm start basis and column generation, decreasing solution times significantly. Lastly, we present computational results using the matheuristic to solve instances of the benchmark suite *LINER-LIB 2012*. The results are the first results for the benchmark suite enforcing a strict weekly frequency for all services. The implementation is quite fast in solving the instances and is able to improve the constructed initial solution with 60-400% yielding profitable networks for twelve out of eighteen solved instances. Additionally a single test case has been constructed to evaluate the matheuristic as a decision support tool, where the initial solution is an existing network and we allow optimization of a subset of the services considering the flow in the entire network. Results are promising for this approach.

The outline of the paper is as follows. In Section 5.2 we review the literature on liner shipping network design. Section 5.3 describes the individual components of our matheuristic. Section 5.4 presents the design of the complete algorithm for the matheuristic. Section 4.5 presents computational results for the matheuristic for LSNDP using the benchmark suite *LINER-LIB 2012* followed by a brief conclusion and perspectives for future work.

5.2 Literature on the LSNP

Brouer et al. (2012) give an introduction to the LSNP focusing on mathematical modelling of the business domain and the introduction of a benchmark suite of LSNP problems.

Christiansen et al. (2004) review the field of operations research within shipping in general and a good introduction to the LSNP may be found in Christiansen et al. (2007). Recently, Kjeldsen (2011) published a classification scheme for routing and scheduling problems within liner shipping reviewing and classifying 24 references. The LSNP was initially studied by Rana and Vickson (1991) as a Mixed Integer Program (MIP) for a multiple container-ship problem without transshipment and where vessels return to the origin node empty. Benders decomposition principle divides the MIP into an integer network subproblem (INS) and a cargo allocation problem (CAS). Results are reported for 10-20 ports and three vessels.

In recent literature several variants of the LSNP have been presented. Fagerholt (2004) developed a model and solution method for a regional carrier along the Norwegian coast. The model assumes the carrier loads at a single port and finds optimal routes of vessels to service the unloading facilities. The problem may be dealt with as a VRP problem, given that a designated depot is known and transshipments are not allowed. The solution method is based on complete enumeration solved by a MIP solver. Similarly, Karlaftis et al. (2009) solved a problem for the region of the Aegean sea using a genetic algorithm. These models do not deal with the important concept of transshipments at multiple ports and the resulting interaction between different services.

The simultaneous ship scheduling and cargo routing problem (SSSCR) by Agarwal and Ergun (2008) is based on a time-space network with each port represented on 7 consecutive weekdays. This construction allows non-simple cycles with multiple visits to a port on different weekdays. Computational results were reported for three different heuristics exploiting the separability of solving the route generation problem and the MCF: a greedy cycle-generation heuristic, a column generation based heuristic and a two-phase Benders decomposition algorithm. Results are reported for 6, 10, 15 and 20 ports with up to 100 ships and 114 demands. The Benders decomposition algorithm is the best performing heuristic of the three, but no optimality gap can be reported for the solutions found as a standard branch-and-bound procedure is invoked for both the Benders, and the column generation algorithm without a complete set of routings. An important limitation of the SSSCR is that it allows transshipments at no cost.

Reinhardt and Pisinger (2011) presented the LSNP for a multiple container ship problem with separate routings for each vessel accounting for transshipment costs between routes. The model allows pseudo-simple cycles, where multiple visits are allowed to one port on a service. A branch-and-cut algorithm is applied to the problem and computational results are reported for 15 ports and up to 6 vessels. Instances of up to 10 ports using 3 vessels are solved to optimality, but the solutions for the larger test instances have a gap of up to 25%.

Alvarez (2009) presented the joint routing and fleet deployment model for the LSNP. The model accounts for transshipment costs and the option of laying up or forward leasing vessels not in use. The model is separable into a service generating problem and a MCF. A service consists of a port call sequence, a number of vessels deployed and an average sailing speed. The overall objective is to maximise the revenue of cargo transported, while considering operational cost of the fleet-, fuel-, transshipment-, and port call-cost. The model is the first to incorporate routings with different speeds in order to optimize on the fuel consumption in the network. Exact solutions are obtained for a six port instance using a MIP solver. Alvarez (2009) describes a tabu search heuristic to solve the problem which is applied to a 120 port instance with a full demand matrix.

Recently, Meng and Wang (2011) presented a mixed integer programming model with the objectives to select among a set of predefined candidate shipping routes, and to select ship deployment to the chosen routes while considering the cargo allocation of full and empty containers regarding the weekly frequency constraint. The model aims at providing decision support on existing routes as well as user-defined candidate routes by minimizing the fixed cost of deployment and the variable costs associated with full and empty containers. The model is solved using CPLEX and numerical results are presented for 60 candidate shipping lines, eight vessel types and 600 commodities.

Brouer et al. (2012) presents a reference model for the LSNDP similar to the model of Alvarez (2009) accounting correctly for transshipments on butterfly routes. A heuristic column generator generating routes using a MIP is used to solve the benchmark suite *LINER-LIB 2012*. The largest instance solved contains 111 ports and 4000 demands.

In light of the literature published on the LSNDP exact solution methods are presently not able to solve large scale instances to optimality. Heuristic methods published are often based on route generation in a branch-and-bound framework with the exception of Alvarez (2009); Brouer et al. (2012), where the overall framework is a local search with route generation as the underlying method for producing a new candidate solution. The present paper aims at improving an existing solution by modifying the current services with insertion and removals of port calls using a multiple quadratic knapsack problem to create an initial solution and a simple method for generating new services to diversify the search. To the best of our knowledge, it is the first heuristic presented for LSNDP using a mixed integer program to define the neighborhood of insertions and removals. The literature identifies a bottleneck in evaluating a candidate solution by solving a large scale arc flow formulation of the MCF using a MIP solver. To increase performance of evaluation we apply a novel method exploiting a warm started delayed column generation algorithm on a path flow formulation of the MCF to evaluate a candidate solution.

5.3 A matheuristic for the liner shipping network design problem

An instance of the liner shipping network design problem consists of:

- A set of ports P .
- A set of demands K , where each demand has an origin, $O_k \in P$, and a destination, $D_k \in P$.
- A set of vessel classes A and a quantity of each vessel class N_a . Each vessel v belongs to a given vessel class a specifying its capacity C_a , minimum and maximum speed limits, bunker consumption per nautical mile and a weekly sailing distance W_d^a . The weekly sailing distance is based on the design speed of the vessel, where fuel consumption is optimized.
- A distance table of the direct distance d_{pq} between all pairs of ports $p, q \in P$.

A solution to the liner shipping network design problem is a set of services S . A service is a cyclic route visiting a set of ports $P' \subseteq P$. The service may be non-simple. The rotation time is the time needed to complete the cyclic route including a day for each port call en route for cargo handling. Depending on the vessel class a a minimum (T_{min}^a) and maximum (T_{max}^a) rotation time in weeks may be defined. It is common in liner shipping to offer a regular service with a weekly frequency. The weekly frequency of port calls is obtained by deploying multiple vessels to a service sailing one week apart. Let n_s^a be the number of vessels of vessel class $a \in A$ deployed to service $s \in S$ to maintain a weekly frequency. A service carries a set of demands $k_s \subseteq K$ either by serving both O_k and D_k or by serving either O_k or D_k and a designated transshipment port G_k valid for transshipping demand $k \in K$.

5.3.1 Mathematical model

A reference model for the LSNDP is provided in Brouer et al. (2012). The variables of this model are rotations defining a service with a vessel class a , the number of vessels deployed and a speed. The rotations generated by the heuristic column generator in Brouer et al. (2012) have a frequency corresponding to the number of vessels, the speed and the distance travelled allowing for (bi)weekly frequencies for vessel classes below 1200 FFE and weekly frequencies for all remaining vessel classes. Additionally this (bi)weekly frequency has a span of 5-7 days frequency for weekly frequency and 10-14 days for bi-weekly frequency. In the work on the LSNDP the question of weekly frequency is disputed as one research trend is towards a planning horizon introducing some

flexibility on the frequency as in Alvarez (2009); Brouer et al. (2012); Reinhardt and Pisinger (2011) whereas Agarwal and Ergun (2008); Plum (2010) enforce strict weekly frequency. In the present paper we focus on a strict weekly frequency for all vessel classes, which gives a more restricted solution space than in the heuristic presented in Brouer et al. (2012). However, the only difference is in the solution space of the rotations and we adhere to the objective function and the constraints of the reference model. The objective accounts for the daily running cost of vessels deployed, the fuel consumption, canal costs and port call cost. A revenue is obtained for the cargo transported and a penalty is incurred for cargo not transported. Finally, a cargo handling costs for loading, unloading and transshipments is charged. Note that we are solving a minimization problem, which means that a negative objective value represents a profit.

5.3.2 Algorithmic overview

The matheuristic creates an initial solution using a greedy construction heuristic. The construction heuristic returns a set of services, S , that are iteratively improved using a *MIP* for each service to indicate a set of port insertions and removal of each individual service. The *MIP* returns a set of moves and the resulting candidate solution is evaluated using a warm started delayed column generation algorithm and a simulated annealing scheme decides whether the candidate is accepted. This phase of the heuristic fine tunes a given solution. The composition of the set of services is important in a high quality solution and we subsequently apply a local search on the composition of the set of services S . Note that the reference model is defined as a minimization problem and a negative objective value is a profitable network. An algorithmic overview is illustrated in figure 5.1.

5.3.3 Generating an initial solution using a greedy construction heuristic

We obtain an initial solution to the LSNDP by constructing a set of services in which we place a set of predefined port calls in order to transport the demand. The construction heuristic transforms an instance of the LSNDP into an instance of the multiple quadratic knapsack problem, where a service corresponds to a knapsack and the items are port calls. It is quadratic in the sense that profit is obtained by adding port pairs to the services in order to transport demand.

The service set problem is based on a subdivision of the available fleet into a set of services S constituted by subsets $S_a \subseteq S$ according to vessel classes. It is desirable to have services of varying duration within an interval $[T_{min}^a; T_{max}^a]$. A random integer $h \in [T_{min}^a; T_{max}^a]$ is selected and a service s with $n_s^a = h$ of h weeks duration (an h -week rotation requires h vessels) is added to the set of services S . Set $v = N_a$. After creation of a new service $s \in S_a$, v is updated by subtracting h from v . This process is repeated until $v \leq T_{max}^a$. If $v \geq T_{min}^a$ the final service is created with $h = v \in [T_{min}^a; T_{max}^a]$ otherwise we add h vessels to the previously created service s' possibly exceeding T_{max}^a . If $n_{s'}^a \geq 2 \cdot T_{min}^a$ we split the service into two services each with $n_s^a = h/2$. After this procedure, a set of services S is defined, a vessel class a and a number of vessels n_s^a is assigned to each service s . The subdivision of the fleet into services means that the initial solution attempts to assign the entire fleet to services.

Next we define a set of port calls to place in the services. Each port can be defined as a main port or an outport. The initial solution is limited to the creation of simple cycles. A port call may be placed only once in each service, but in m_p services. Outports have $m_p = 3$, whereas main ports have $m_p = 10$. There is no constraint requiring all port calls placed in the set of services.

The profit of transporting a demand from port $i \in P$ to $j \in P$ is the revenue r_k obtained by the transport subtracted the loading and unloading cost (c_l^i and c_u^i respectively) of the container en route. A demand transported with no transshipments will have net revenue $\rho_{O_k D_k} = (r_k - c_l^{O_k} - c_u^{D_k})$ for one unit of k . As described in the introduction more than 50% of the demands are transshipped resulting in a MCF. The multiple quadratic knapsack problem does not consider this MCF, and in order to model transshipments the demand matrix is transformed such that each demand is represented by a direct demand and a demand transshipped at a designated

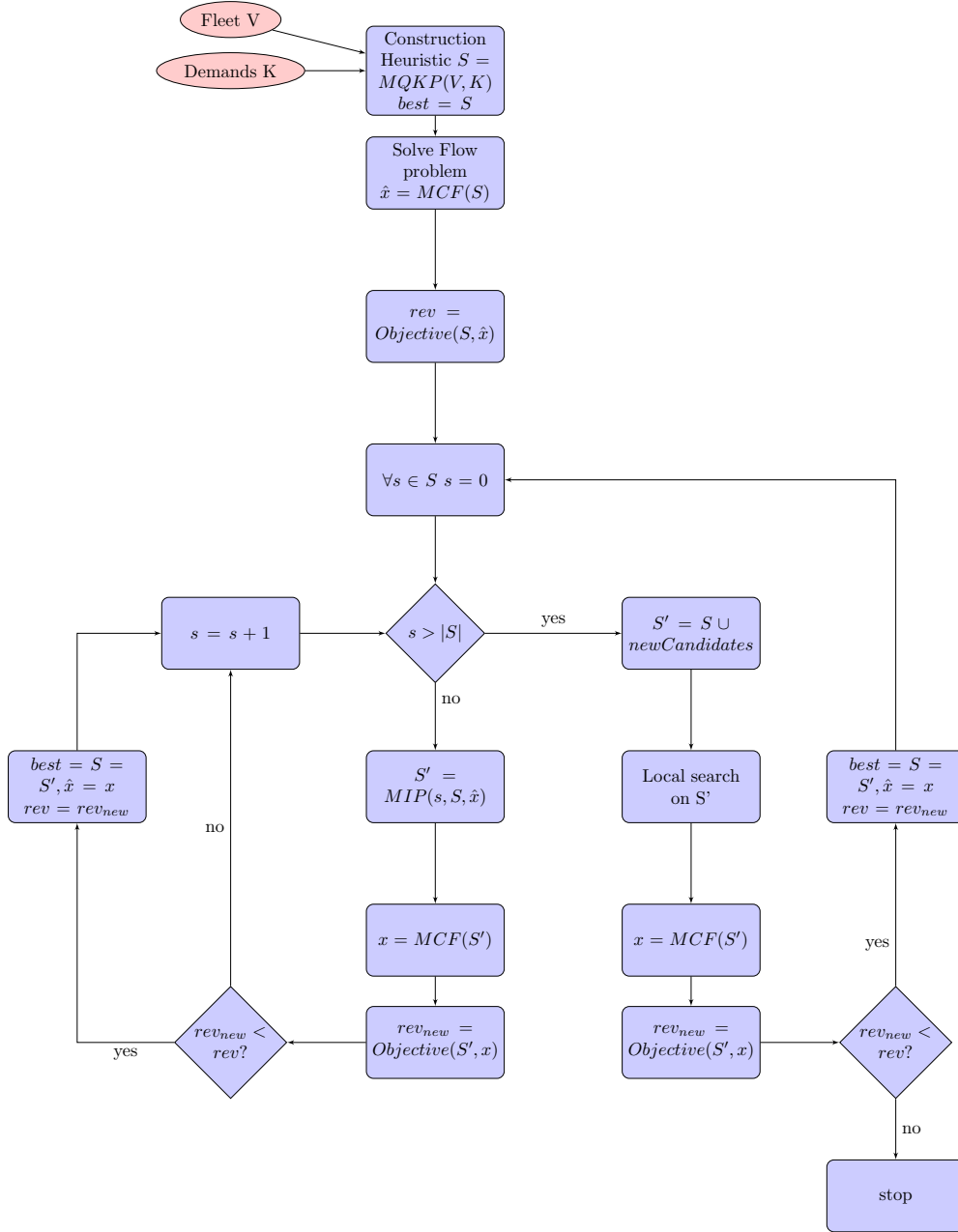


Figure 5.1: A flow chart of the matheuristic.

transshipment port G_k , where $\rho_{O_k G_k} = (\frac{1}{2}r_k - c_l^{O_k} - c_u^{G_k})$ and $\rho_{G_k D_k} = (\frac{1}{2}r_k - c_l^{G_k} - c_u^{D_k})$. This is a simplifying assumption fixing a single transshipment port for each demand to incorporate interaction between services in the construction heuristic. The subsequent improvement heuristic will have no restrictions on transshipment facilities. A port call cost, c_p^a , is associated with a port call depending on the vessel class and a sailing cost is associated with each port pair, c_{pq}^a .

The construction heuristic is a greedy parallel insertion heuristic. The services are seeded with a random number $l \in \{1; 3\}$ of ports $p \in P$. The seeding is either by random or by selecting a port $p \in P$ and a transshipment port $q \in P$ matched to p . The construction heuristic is based on parallel insertion by a *football teaming principle* i.e. the services take turn at choosing the next port to call. We apply parallel insertion in order to disperse the attractive port call

combinations throughout the network. A greedy choice of the most revenue generating port call is made between all feasible port calls with regards to route duration. Feasibility of a given port call is estimated using best insertion in order to respect the weekly frequency constraint, requiring the distance of a route $D_s \leq W_d^{a(s)}(n_s^{a(s)} - (\frac{|P_s|}{7}))$, where $|P_s|$ are the number of port calls in service s , $a(s)$ is the vessel class $a \in A$ deployed to service $s \in S$. The number $\frac{|P_s|}{7}$ accounts for the accumulated port stay of the service, where 24 hours are used in each port of call effectively decreasing the time left for actually sailing a round trip. The actual routing with regards to distance and capacity utilization is improved using a local search based on simulated annealing and two-opt after assignment of port calls to services by the greedy construction heuristic. The initial solution may have unplaced port calls and excess vessels for services s , where the distance of a route allows this ($D_s \leq W_d^{a(s)}(n_s^{a(s)} - (\frac{|P_s|}{7}) - 1)$). Port calls as well as vessels may be included in the solution of the subsequent improvement heuristic. Finally, we apply standard column generation to the MCF of transporting the cargo on the resulting liner shipping network of the initial solution. The solution of the MCF is used to calculate the estimation function values of the improvement heuristic.

5.3.4 Improvement heuristic

Given a solution to the LSNDP x' with services S' serving demands $K' \subseteq K$ we introduce an integer program to estimate the effect of removing and adding port calls. We define:

- P^s is the set of nodes in the service $s \in S'$.
- $N^s \subset P \setminus P^s$ is the set of neighbors of a service $s \in S'$ defined as nodes within a certain geographical distance of nodes in P^s .

We have the variables:

- $\lambda_i = 1$ if item $i \in P^s$ is removed from service $s \in S'$, 0 otherwise.
- $\gamma_i = 1$ if item $i \in N^s$ is inserted in service $s \in S'$, 0 otherwise.
- $\omega_s \in \mathbb{Z}^+$ an integer variable indicating the number of vessels service s is expanded with. ω_s can be negative if less vessels are needed after removal of a port call.

We want to make an integer program that removes and inserts port calls in S' , while considering an estimation of the duration of each service (the fleet deployment) and an estimation of the alternative flow of demands arising, when we remove/insert several port calls from/to S' . Routing the cargo is a MCF, but we cannot afford to evaluate the MCF in its entirety and hence we make some simplifying assumptions about rerouting the flow.

Estimation functions for the distance travelled by each service $s \in S'$

When inserting a port call the estimated distance increase is calculated by use of a best insertion heuristic. For each service $s \in S'$ we calculate the distance increase Δ_i^s for each $i \in N^s$. Likewise we calculate the decrease of distance Γ_i^s for every $i \in P^s$. For modelling the distance in-/decrease of insertions/removals we define the following constants and sets:

Δ_i^s : estimated distance increase for inserting item i in service $s \in S$ according to a best insertion method.

Γ_i^s : estimated distance decrease for removing item i from service $s \in S$ joining its predecessor with its successor.

E_s : set of edges used by the Hamiltonian cycle in service $s \in S$.

$D(s)$: current distance of the Hamiltonian cycle in service $s \in S$.

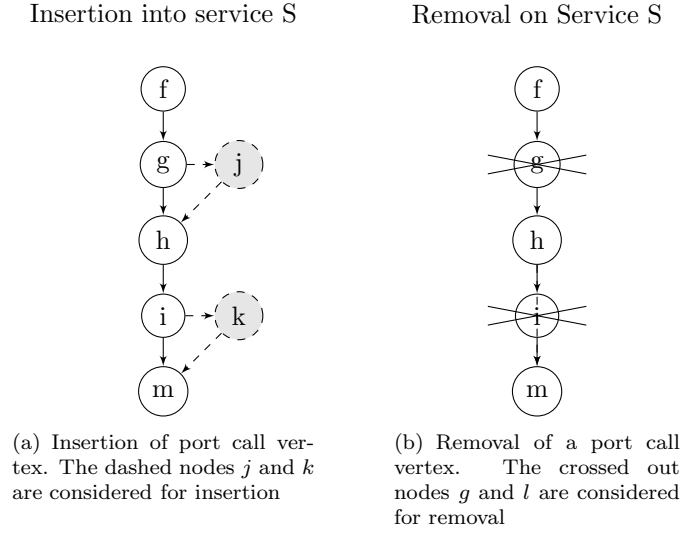


Figure 5.2: The neighborhood consists of a combination of insertion and removal moves

M_a : number of undeployed vessels of class a in the current service set S' .

n_s^a : number of deployed vessels of class a to service $s \in S'$.

C_v^a : cost of deploying a vessel of type $a \in A$.

Estimation functions for the change in the multicommodity flow

Whenever a MIP is solved for some $s \in S'$ we estimate the effect on the flow in the network by solving shortest path problems on the residual capacity of the network. The quality of the flow solution depends on:

1. The number of transshipments performed overall in the network.
2. The capacity installed compared to the demand for flow.

We define the following estimation functions:

$\Theta(i)$: estimated value of inserting a node $i \in N^s$ in the best insertion position identified when calculating the distance.

$\Upsilon(i)$: estimated value of removing a node $i \in P^s$.

$\Psi(i)$: estimated value of reinserting a node $i \in P^s$ by best insertion limited to insertions two port calls away from the current position of i in s .

Graph topology

In order to estimate the change of the network flow a graph $G = (V, E)$ defined by the residual capacity is constructed, representing the solution x' with services S' and commodity allocation K' mapped onto the network by solving the MCF on S' . Let:

- $|s|$ denote the number of unique ports in s and let $|P^s| = m$ denote the number of port calls in a rotation r^s for s , $|P^s| = m \geq |s|$.
- r^s be a rotation defined by the port sequence $p_1^s, p_2^s, \dots, p_m^s$.
- V_p be the set of port vertices.

- V_{r^s} be a set of vertices representing the port call sequence $p_1^s, p_2^s, \dots, p_m^s$ for rotation r^s .
- $V_r = \bigcup_s V_{r^s}$ be the set of rotation vertices representing all port calls by all rotations.
- $V = V_p \cup V_r$ be the set of vertices.
- $E = E_l \cup E_d \cup E_v \cup E_t$ be the set of edges, where
 1. $E_l = \{(p, v) | p \in V_p, v \in V_{r^s}\}$ is the set of load edges representing a departure from port p to the rotation r^s .
 2. $E_d = \{(v, p) | v \in V_{r^s}, p \in V_p\}$ is the set of discharge edges representing an arrival at port p from the rotation r^s .
 3. $E_t = \{(v, u) | v \in V_{r^s}, u \in V_{r^{s'}}\}$ is the set of transshipment edges representing a transshipment between rotations r^s and $r^{s'}$ defined for every (v, u) where $p(v) = p(u)$, where $p(w)$ denotes the physical port of the port call w .
 4. $E_v = \{(v, u) | v, u \in V_{r^s}, v = p_h^s, u = p_{((h+1) \bmod m)}^s\}$ is the set of voyage edges representing a voyage between two consecutive port calls in r^s defined $\forall h \in \{1, 2, \dots, m\}$.
- C_e be the capacity of edge $e \in E$, where $C_e = \infty$ for $e \in E_l \cup E_d \cup E_t$ and $C_e, e \in E_v$ be the residual capacity of edge e after flow assignment of the MCF onto S' .
- c_e be the edge cost, where $c_e = 0, e \in E_v$ (as the cost is on the vessel) and let $c_e = c_l^p, e \in E_l$ and $c_e = c_u^p, e \in E_d$ be the cargo handling cost of loading or unloading a container at port $p \in V_p$, where p is either the source or the target of the edge respectively. For the transshipment edges $c_e = c_t^p$, where p is the physical port of the port calls of the source and the target of edge e .

An example of a hub and spoke network with one hub, C , and five spokes (A, B, D, E, F) and 3 rotations is illustrated in Figure 5.3. The rotations are $A \rightarrow B \rightarrow C \rightarrow A$ (index 1), $C \rightarrow A \rightarrow C \rightarrow D \rightarrow C$ (index 2), $F \rightarrow E \rightarrow C \rightarrow F$ (index 3). Each rotation vertex has a load and a discharge edge to the physical port and a voyage edge to the next port en route. A demand from A to F transships at C using rotations 2 and 3: $A \rightarrow A2 \rightarrow C2 \rightarrow C3 \rightarrow F3 \rightarrow F$ (edge colours green and blue in Figure 5.3) or using rotations with indices 1 and 3: $A \rightarrow A1 \rightarrow B1 \rightarrow C1 \rightarrow C3 \rightarrow F3 \rightarrow F$ (edge colours red and blue in Figure 5.3). The load/unload cost and the transshipment cost is accounted for by the path cost. The load/unload is distinct from the transshipment as some ports have a different(lower) cost for transshipment than for a load combined with an unload. Note, that multiple visits to a port on a service results in multiple vertices. To adhere to the reference model a single port is allowed to be called twice.

The estimated value of insertion - $\Theta(i)$

When we insert a port call vertex $i^s \in N^s$ with corresponding port vertex $i \in V_p$ in the position between nodes h^s and l^s the demands of the set $K_i = \{k \in K | i = O_k \vee i = D_k\}$ become eligible for transport using service $s \in S'$. Solving a shortest path problem on G' where $V_{r^s}' \cup \{i^s\}$, and $E_v' = E_v \setminus \{(h^s l^s)\} \cup \{(h^s i^s), (i^s l^s)\}$, $E_l' = E_l \cup \{(i^s)\}$, $E_d' = E_d \cup \{(i^s)\}$ will identify for each $k \in K_i$ whether there is an (improved) path for k in G' in terms of transshipment costs (TC), the increase of revenue in demand transported (RK) and the capacity available. The estimated value $\Theta(i)$ should account for in-/decrease in transshipment cost, in-/decreased revenue of the flow, and increase in port call cost: $\Theta(i) = TC(G', K_i) - TC(G, K_i) + RK(G') - RK(G) - c_i^s$.

The estimated value of removal - $\Upsilon(i)$

When a port call vertex $i^s \in P^s$ is removed between nodes h^s and l^s , commodities of the set K_i transported on s must be rerouted or omitted. Define $K_i^s = \{k \in K_i | k \text{ is transported on } s\}$. $\Upsilon(i)$ estimates rerouting K_i^s in the remaining network by solving a shortest path problem on

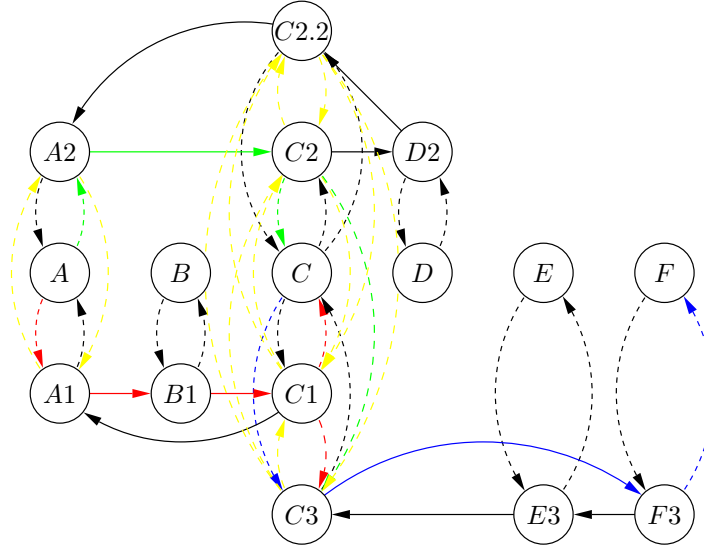


Figure 5.3: Example network with 6 ports (A, B, C, D, E, F) and 3 rotations. C is the linking hub. Continuous arcs are voyage edges E_v , whereas dotted arcs are load, discharge and transshipment edges $E_l \cup E_d \cup E_t$. E_t are yellow to set them apart from load/unload edges. The red (A, A1, B1, C1, C)- and green (A, A2, C2, C) paths are two distinct ways to travel from A to C. The blue (C, C3, F3, F) path is the only path from C to F.

$G' = (V_{r^s} \setminus \{i^s\}, E_v \setminus \{(h^s i^s), (i^s l^s)\} \cup \{(h^s l^s)\})$. G' will identify for each $k \in K_i^s$ whether there is an alternative path in the network. The estimated value $\Upsilon(i)$ should account for the in-/decrease in transshipment cost TC for each commodity $k \in K_i^s$ rerouted in G' , and the decrease of revenue flow RK for omitted cargo and the decrease in port call cost.

$$\Upsilon(i) = TC(G', K_i) - TC(G, K_i) + RK(G') - RK(G) - c_i^s.$$

Lock sets

The estimation functions are used to make a MIP for a single service with the remaining services fixed. A solution to the MIP may result in several insertions and removals referred to as a *move* in the following. The estimation functions are based on performing a particular move without consideration of additional removals/insertions. In order to reduce the error of the estimation functions we define *lock sets* of a move constraining insertions/removals on port calls related to a move.

When inserting a port call i , a set of new commodities K_i may be transported. The origins and destinations of $k \in K_i$ should not be removed. The estimation function relies on the residual capacity of the remaining network. Insertions before bottlenecks introduced by the routing of K_i should be avoided. We define the set of *Insertion locks* on inserting $i \in N^s$ as $L(i^+)$. $L(i^+)$ place a lock on removal of origin/destination nodes ($i \in P^s$) for $k \in K_i$, and lock on insertion of nodes ($i \in N^s$) with best insertion position before bottlenecks introduced by routing K_i .

The total number of removals from a service is constrained to F_s . F_s is input by the user or dependent on the number of port calls on a service. F_s should be small to reduce the error of the distance decrease function Γ_i^s .

5.3.5 MIP formulation

The following MIP optimizes a single service and suggests a set of removals and insertions of port calls. The function $a(s)$ returns the vessel class assigned to service s .

$$\max \sum_{i \in N^s} \Theta(i) \gamma_i + \sum_{i \in P^s} \Upsilon(i) \lambda_i - C_v^{a(s)} \omega_s \quad (5.1)$$

$$\text{subject to: } D(s) + \sum_{i \in N^s} \Delta_i^s \gamma_i - \sum_{i \in P^s} \Gamma_i^s \lambda_i \leq W_d^{a(s)} \left(n_s^a - \frac{|P_s| + \sum_{i \in N^s} \gamma_i - \sum_{i \in P^s} \lambda_i}{7} + \omega_s \right) \quad (5.2)$$

$$\omega_s \leq M_{a(s)} \quad (5.3)$$

$$\sum_{i \in P^s} \lambda_i \leq F_s \quad (5.4)$$

$$\sum_{j \in L(i^+)} \gamma_j + \lambda_j - 2|L(i^+)|(1 - \gamma_i) \leq 0 \quad \forall i \in N^s \quad (5.5)$$

$$\lambda_i \in \{0, 1\} \quad \forall i \in P^s \quad (5.6)$$

$$\gamma_i \in \{0, 1\} \quad \forall i \in N^s \quad (5.7)$$

$$\omega_s \in \mathcal{Z} \quad (5.8)$$

Sets

P^s	The set of nodes in the service s .
N^s	The set of neighbors of service s .
$L(i^+)$	Lock set of inserting i in s .

Constants

$C_v^{a(s)}$	The weekly cost of a vessel from vessel class a assigned to service s .
$M_{a(s)}$	The number of undeployed vessels of vessel class a assigned to service s in the current solution.
$W_d^{a(s)}$	The weekly sailing distance of a vessel of class a at design speed.
F_s	number of removals allowed on each service

Variables

λ_i	1 iff $i \in P^s$ is removed from service s , 0 otherwise.
γ_i	1 if item $i \in N^s$ is inserted in service s , 0 otherwise.
$\omega_s \in \mathcal{Z}^+$	The number of vessels service s is expanded with.

Functions

Δ_i^s	Estimated distance increase if i is inserted in s .
Γ_i^s	Estimated Distance decrease for removing item i from s .
$\Theta(i)$	The estimated value of inserting i in s .
$\Upsilon(j)$	The estimated value of removing j from s

Table 5.2: Overview of Sets, variables, and functions used in the MIP

The objective function (5.1) maximises the benefit obtained from removing and inserting several port calls accounting for the estimated change of revenue, transshipment cost, port call cost and fleet cost. The number of vessels needed to maintain weekly frequency on the service after insertion/removal is estimated in Constraints (5.2) accounting for the duration of the port stay given insertions/removals. Constraints (5.3) ensure that the solution does not exceed the available fleet of vessels. The set of nodes that are affected by the insertion move are fixed by Constraints (5.5). Constraints (5.4) ensure that we can only remove F_s nodes from the service.

To diversify the search using the MIP the geographical distance to the ports included in the neighborhood N_s is increased for every iteration, where no improvement is found. As the ports on a service are often geographically close to each other some ports in the service have neighboring ports in common. For this reason the neighborhood size varies a lot for the service even when increasing the geographical distance. The increase in geographical distance is inspired by Variable

Neighborhood Search (VNS)(Mladenović and Hansen, 1997), where the neighborhood depth is increased iteratively.

We introduce a number of taboos in each service to avoid cycling between solutions. This is to ensure that ports inserted in the previous iteration cannot be removed in the subsequent MIP for a specific service. Likewise, ports removed in the last iteration are not considered for reinsertion in the following iteration.

5.3.6 Reinsertion neighborhood

It is important to be able to construct non-simple routes (butterfly rotations), that revisit a single port twice. Butterfly routes are very important to utilize the capacity efficiently in a liner shipping network. A reinsertion neighborhood is defined to construct butterfly routes by evaluating reinsertion of port calls on every service in S . The reinsertion neighborhood loops through the services in S and identifies the most promising port call to reinsert based upon a rating. The method identifies every port call, where the outgoing voyage arc has no residual capacity. Each of the candidate port calls on a service are rated based on the residual demand. A score of 1 is obtained for each residual demand originating at the candidate port and an additional score is obtained if the destination of the commodity is found on the service. The port obtains an additional score of 3 if the port is a designated hub port. Based on the rating the port with the highest rating is reinserted using the best insertion heuristic requiring the reinsertion to be a minimum of one port call away from the original port call. The reinsertion is immediately evaluated and accepted if it improves the objective value or increases the amount of cargo transported. An overview of the reinsertion neighborhood is found in Algorithm 1

Algorithm 1 *EvaluateReinsertion*(S', x', \tilde{K})

Require: A solution S of the LSNDP (S, \tilde{K}, x)

```

1:  $\hat{S} \leftarrow S'$ 
2:  $\hat{x} \leftarrow x'$ 
3: for all  $s \in S'$  do
4:    $Candidate \leftarrow \emptyset$ 
5:   for all  $p_i^s \in s$  do
6:     if  $flow(p_i^s, p_{i+1}^s) - Capacity(p_i^s, p_{i+1}^s) = 0$  then
7:        $Candidate \leftarrow p_i^s$ 
8:        $rating_{p_i^s} \leftarrow AssignRating(\tilde{K}, p_i^s, S)$ 
9:     end if
10:  end for
11:  if  $Candidate \neq \emptyset$  then
12:     $R \leftarrow MaxRating(Candidate)$ 
13:     $\tilde{S} \leftarrow ApplyReinsertion(R, s)$ 
14:     $\bar{x} \leftarrow \Delta MCF(\tilde{S}, K)$ 
15:  end if
16:  if  $obj(\bar{x}) \leq obj(x') \vee trans(\bar{x}) > trans(x')$  then
17:     $x' \leftarrow \bar{x}$ 
18:     $S' \leftarrow \tilde{S}$ 
19:     $\tilde{K} = CalcResidualDemand(S', x')$ 
20:  end if
21: end for
22: return ( $S', x'$ )

```

5.3.7 Local search

The solution space of an instance of the LSNDP has several sub-neighborhoods that must be considered in a search for a high quality solution. The main sub-neighborhoods are listed below:

1. The individual port calls on each service.
2. The selection of the butterfly port.
3. Transshipment points.
4. Deployment of a number of vessels to a service (i.e. the duration assigned)
5. Average speed of the vessels on a service.
6. The composition of the services.
7. The number of services in the solution.
8. Matching vessel classes to services.

The MIP and the reinsertion neighborhood target sub-neighborhood 1, 2, 3, and 4. In the application of a move sub-neighborhood 5 is considered as well. An adjustment of the number of vessels is performed relative to the feasibility and cost of assigning one less or one more vessel to the service. If it is cheaper and feasible to add/subtract a vessel from a service, this optimization is performed, when applying the move. This procedure also adjusts the speed of a service, although the design speed will be the cheapest choice if it can be matched with the duration and the number of vessels assigned to a service.

A weakness of the MIP neighborhood is that it is not able to adjust the number of services. The number of services in the initial solution of the construction heuristic is not likely to be optimal. At the same time the composition of services is not considered for example if two services are roughly identical and therefore both underutilized with regards to their capacity.

Therefore, a local search is performed on the set of services after every fourth loop of the MIP neighborhood application to the set of services.

The local search rates every service according to its average utilization of the capacity on all voyage edges. In increasing order of utilization removing a service is evaluated. Up to two services are removed using this procedure. The limitation is imposed to decrease the size of the move. The residual demand is updated for the solution with services removed and a procedure to add new services based on the residual demand is invoked. A single service is added for every vessel class with undeployed vessels with the largest matching residual demand respecting feasibility of the capacity and duration of serving the demand with the vessels available.

Lastly, the MIP sub-neighborhood cannot assign a different vessel class to a service for example a service which is fully utilized and in general needs to be upgraded to a larger vessel class with more capacity and likewise a service with a medium average utilization may become more profitable by assigning a smaller vessel class to the service. In the current implementation we do not consider swapping vessel classes between services although it would be very interesting to investigate the performance of such a neighborhood. The neighborhood would require extensive analysis to find good candidate services for swapping vessel classes and would involve reflowing the cargo upon each swap to evaluate the move. It is hence not trivial to implement.

5.4 Algorithm

Algorithm 2 gives an overview of the matheuristic. The initial solution is constructed by the greedy parallel insertion *GreedyLSNDP(I)* of an instance I in line 1. The initial solution does not contain exact evaluation of the flow and hence we explicitly solve the resulting MCF in line 2. The problem $MCF(S', K)$ is solved using standard column generation to construct a graph of

Algorithm 2 MatHeuristic(I)**Require:** An Instance I of the LSNDP (S, P, A, D, K)

```

1:  $S' \leftarrow GreedyLSNDP(I)$ 
2:  $x' \leftarrow MCF(S', K)$ 
3:  $iter \leftarrow 0$ 
4:  $temp \leftarrow temp_0$ 
5:  $IMPROVE \leftarrow true$ 
6: while  $(IMPROVE \wedge (temp > 0))$  do
7:    $IMPROVE \leftarrow false$ 
8:   for all  $s \in S'$  do
9:     if  $obj(x') \leq obj(\hat{x})$  then
10:        $SaveBest(S', x')$ 
11:     end if
12:     for all  $i \in N^s$  do
13:        $(L(i^+), \Theta(i)) \leftarrow CalcInsert(i, S')$ 
14:     end for
15:     for all  $i \in P^s$  do
16:        $\Upsilon(i) \leftarrow CalcRemoval(i, S')$ 
17:     end for
18:      $MIP \leftarrow constructMIP(s, P^s, N^s, \Theta, \Upsilon, \Psi, \bigcup_{i \in N^s \cup P^s} L(i^+), \bigcup_{i \in P^s} L(i^2))$ 
19:      $M \leftarrow solve(MIP)$ 
20:     if  $M \neq \emptyset$  then
21:        $\bar{S} \leftarrow ApplyMoves(M, L(i_-^s), s)$ 
22:        $\bar{x} \leftarrow \Delta MCF(S', K)$ 
23:     end if
24:     if  $(obj(\bar{x}) \leq obj(x')) \vee (exp(\frac{obj(x') - obj(\bar{x})}{temp}) > random[0, 1])$  then
25:        $x' \leftarrow \bar{x}$ 
26:        $S' \leftarrow \bar{S}$ 
27:        $IMPROVE \leftarrow true$ 
28:     end if
29:      $temp \leftarrow temp \cdot 0.95$ 
30:   end for
31:   if  $iter \bmod 2 = 0$  then
32:      $(S', x') \leftarrow EvaluateReinsertion(S', x')$ 
33:   end if
34:   if  $iter \bmod 5 = 0$  then
35:      $S' \leftarrow LocalSearch(S', x')$ 
36:      $temp \leftarrow temp * 10$ 
37:   end if
38: end while
39: return  $(\hat{S}, \hat{x})$ 

```

the residual capacity and mapping the current flow to specific services. In every iteration of the for loop the solution is evaluated according to the best found solution (line 9).

The improvement heuristic loops over the set of services S . The estimation functions and lock sets for s are calculated in lines 12-17. The MIP (5.1)-(5.8) for s is constructed and solved in lines 18-19.

The solution is evaluated by resolving the new MCF in line 22. $\Delta MCF(S', K)$ is a column generation algorithm for the MCF using a warm start basis. The basis consists of all commodities and services not directly affected by the moves identified by the MIP. The algorithm $\Delta MCF(S', K)$ is expected to decrease solution times, but the performance will depend on the number of commodi-

ties affected by the move and also the number of moves applied. If the solution is improved the new solution is saved in line 24-26 before the next MIP is calculated for the following $s \in S'$. If the solution is not improved a simulated annealing scheme is invoked, to accept worse solutions with a decreasing probability as the search progresses in lines 24-27. The simulated annealing scheme should help the algorithm escape from local minima. In every second loop over the set of services S a reinsertion neighborhood is invoked in order to evaluate the construction of butterfly services in line 32. In every fourth loop a local search is performed on the set of services evaluating the removal of underutilized services and adding new services based on the residual demand and the excess fleet (line 35). The algorithm terminates, when there is no improvement through a loop or the temperature of the simulated annealing is zero. The best found solution is returned in line 39.

5.5 Computational Results

The matheuristic was tested on the benchmark suite *LINER-LIB 2012* presented in Brouer et al. (2012). Table 5.3 gives an overview of the instances, which may be found at <http://www.or.man.dtu.dk/English/research/>

Category	Instance and description	Ports	Dmnds
Single hub instances	Baltic Baltic sea with Bremerhaven as hub	12	22
	WAF West Africa with Algeciras as hub to West African ports	19	38
Multi hub instances	Mediterranean Mediterranean with Algeciras, Tangier, Gioia Tauro as hub	39	369
Trade lane instances	Pacific (Asia-US West)	45	722
	AsiaEurope Europe, Middle East and Far east regions	111	4000
World instances	Small 47 Main ports worldwide identified by Maersk Line	47	1764
	Large 197 ports - the majority of ports serviced directly by Maersk Line	197	9630

Table 5.3: The instances of the benchmark suite with indication of the number of ports and the number of distinct origin-destination pairs. The instances may be found at <http://www.or.man.dtu.dk/English/research/instances>

The matheuristic is coded in C++ and run on a linux system with a *Intel(R) Xeon(R) X5550* CPU at *2.67GHz* and *24 GB* RAM.

The following sections analyze the performance and the results of the algorithm. The first section analyzes the execution time required to solve the MIP neighborhood with growing instance size and the speed up of using delta column generation, when evaluating the neighborhood. The next section describes the results seen in relation to the results presented in Brouer et al. (2012) because our solution space is more restricted due to a required weekly frequency in the matheuristic for LSNDP. This makes it difficult to compare results. The final section presents the computational results for the benchmark suite *LINER-LIB 2012* with a weekly frequency and discusses the results obtained.

5.6 Performance of delta column generator and MIP neighborhood

In this section the performance of solving the MIPs to decide on the moves made to each service is evaluated. Secondly, the performance of evaluating a given solution for the flow transported using a warm started delayed column generation algorithm to solve the underlying MCF problem is presented.

In Table 5.4 the performance of the MIP neighborhoods are reported. The focus is on the execution time of building the MIP in terms of calculating the estimation functions and solving

the resulting MIP. The table shows that calculating the estimation functions and building the MIP is very fast. At the same time the MIPs are very small and solved fast both for small, medium and large cases. The growth in the neighborhoods is hence a linear growth with the number of services that increase with the instance size. The aim of the heuristic was exactly to achieve a linear growth in evaluating the neighborhood of services, such that it is fast to decide on the best moves for each service with regards to inserting and removing a number of port calls for each service considering the network flow as a whole.

Instance	av. time build (sec.)	av. time MIP (sec.)	av. vars	av. rows
Baltic	0.00012	0.00748	9.26	10.53
WAF	0.00036	0.00946	11.19	12.27
Mediterranean	0.00100	0.00972	19.32	19.81
Pacific	0.00390	0.00902	19.11	20.44
WorldSmall	0.01260	0.00600	17.41	18.72
AsiaEurope	0.02100	0.00820	19.93	21.33

Table 5.4: Av. time build: Time to build MIP (estimation functions), **av. time MIP:** Time to solve MIP , **av. vars:** Average number of variables (binary), **av. rows:** average number of rows.

In Table 5.5 the performance of an average run of the delta column generator is reported. The delta column generator is designed to exploit that inserting/removing a few port calls only invalidate a small fraction of the current optimal basis for the network. Therefore, we are able to reuse the optimal basis to a large extent. The table shows the time to solve the initial full multicommodity flow problem and the subsequent columns show the average solution time for the delta column generator as the maximal, minimum and average time throughout a single run of the algorithm for each instance. The result show a speed up of a factor 2-10 on average for evaluating a given solution with regards to the transported commodities and their transshipment costs.

Instance	full sec.	max sec.	min sec.	av. sec	av. speedup	max speedup
Baltic	0.0040	0.004	0.000	0.0015	2.67	—
WAF	0.0240	0.008	0.004	0.0045	5.33	6
Mediterranean	0.3760	0.050	0.016	0.0314	11.97	23
Pacific	3.6122	1.528	0.072	0.7178	5.03	50
WorldSmall	24.5400	12.316	0.024	2.7800	8.80	1022
AsiaEurope	147.9612	60.667	0.836	19.7500	7.49	177

Table 5.5: full sec.: Time to solve full lp, **max sec.:** maximum time for a warm start , **min sec.:** min sec for a warm start, **av. sec:** average time for warm start. **av. speedup** is the solution time for solving the full MCF divided by the av. seconds to solve the warm start. **max speedup** is the solution time for the full MCF divided by the minimum seconds to solve the warm started solution.

5.7 Comparing results

The problem solved with the matheuristic for LSNDP uses the reference model presented in Brouer et al. (2012) and adheres to the same set of general constraints and the same objective function. However, the routes generated by the matheuristic are restricted to have strict weekly frequencies for all vessel classes, whereas the heuristic column generator creates routes with both weekly and even biweekly frequencies for vessel classes under 1200 FFE. The rotations are allowed to deviate from a strict weekly frequency and may call as often as every five days for weekly frequencies. Vessel classes below 1200 FFE are allowed to have biweekly frequencies deviating to call as often as every ten days. This means that the network generated with the heuristic column generator has a less restricted solution space and may deploy the capacity in a different manner than the matheuristic for the LSNDP is able to. A direct comparison with the solutions and objective values

for the heuristic column generator is not meaningful as the best solutions found are not feasible solutions for the matheuristic. As a result the overall revenue obtained for the matheuristic and the reference model with strict weekly frequencies are expected to be lower. Please note, that we are solving a minimization problem and a negative objective value represents a profit. The computational results presented here are the first results requiring strict weekly frequency using a restricted version of the reference model to solve *LINER-LIB 2012*.

Instance	Method	Best Cost	trans %	Depl. %	Time
Baltic	Best weekly freq.	$-4.98 \cdot 10^6$	97.1	100	–
Baltic	Best weekly freq. + matheuristic	$-6.78 \cdot 10^6$	96.76	88.88	4.46
Baltic	MQKP+matheuristic (best)	$-8.60 \cdot 10^6$	97.23	88.89	2.05
Baltic	MQKP+matheuristic (Avg.)	$-1.19 \cdot 10^6$	94.77	88.89	4.17
Pacific	Best weekly freq.	$-6.64 \cdot 10^6$	96.80	98.10	–
Pacific	Best weekly freq. + matheuristic	$-3.57 \cdot 10^7$	96.70	93.5	623.45
Pacific	MQKP+matheuristic (best)	$1.03 \cdot 10^8$	96.02	94.39	825.96
Pacific	MQKP+matheuristic (Avg.)	$1.91 \cdot 10^8$	84.18	87.47	567.84

Table 5.6: Comparing the heuristic column generators best solution for the Baltic and Pacific case with an adjusted increased fleet to enforce strict weekly frequency. **Instance** denotes the instance. **Method** describes, which initial solution is used, and whether the matheuristic is invoked. **Best cost** denotes the best objective value found. When calculating with the construction heuristic as initial solution ten runs were performed and the best as well as the average solution is reported. **trans %** denotes the percentage of total cargo transported and **Depl. %** the percentage of the fleet deployed. **Time** denotes the CPU time in seconds.

In an attempt to compare the performance of the algorithms the best solutions from the Baltic case and the Pacific case have been altered to respect a weekly frequency by increasing the number of vessels in the fleet to meet this requirement. It is obvious, that the heuristic column generator could find different and possibly better solutions enforcing strict weekly frequency with the increased fleet. There will be excess capacity on the routes with biweekly frequency, which for the Baltic case is all vessel classes in the instance. In the Pacific case it represents half the vessel classes and less than half the total number of vessels (36%).

The Baltic case has been chosen as a reference because it is a small case and the matheuristic for LSNDP is relatively stable at finding good solutions for this instance, whereas the Pacific case is more complex and less influenced by biweekly frequency. The revenues and the fleet available, makes it hard to obtain a revenue and the matheuristic consistently performs poorly on this case.

The comparison takes the services from the best solution for the base case using the heuristic column generator adjusting the number of vessels to meet a weekly frequency requirement (Best weekly freq. in Table 5.6). They are used as initial solution and the matheuristic is able to improve upon the solution in its subsequent optimization for both instances. These results are compared against the same fleet using the construction heuristic. The matheuristic is able to find better solutions using the construction heuristic as opposed to the generated solutions for the Baltic case, but cannot compete on the Pacific case using the construction heuristic. In the Pacific case results show great variance in the revenue obtained and it is clear that this case is very dependant on the quality of the initial solution. The more complex instances have a solution space with many sub-neighborhoods and it seems that the matheuristic gets trapped in a local minimum, where it is not able to improve further or access more promising areas of the solution space. The results are presented in Table 5.6.

5.8 Algorithmic performance and results for *LINER-LIB 2012*

The results of testing the matheuristic on *LINER-LIB 2012* is presented in Table 5.8. For each instance ten replications with random seeds have been run. Randomness affects the initial solution constructed by the MQKP method and also the simulated annealing scheme incorporated in the subsequent improvement heuristic.

The algorithm has been set to terminate after the time limits imposed in Brouer et al. (2012) as stated in Table 5.7.

Baltic	WAF	Mediterranean	Pacific	AsiaEurope	WorldSmall
300	900	1200	3600	14400	10800

Table 5.7: Time limits imposed on each instance. the time is in CPU seconds.

The results are the first presented for a strict weekly frequency on rotations and the revenue is generally lower than the results presented in Brouer et al. (2012) as expected due to different frequency requirements. Results show a large deviation between the best found solutions and the average solution for most cases. The heuristic is very dependant on the quality of the initial solution, which deteriorates with instance size, as it is harder to get the right composition and fleet deployment as instance sizes grow. Furthermore, the solution space is complex with many sub-neighborhoods and the algorithm seems to get easily trapped in a local minimum. In the Mediterranean case the fleet deployment is very low and there is rejected cargo. It seems that the algorithm is not able to properly deploy the excess vessels to capture the residual demand. In general the fleet deployment is high, meaning that requiring a weekly frequency may not give sufficient capacity to transport the entire demand or the demands are not profitable to transport given the fleet /service configuration. The algorithm is fast for all instances but the AsiaEurope instance, which is the only one terminating due to the time limit imposed of 14400 seconds. The log files indicate that the loop structure over the entire set of S as illustrated in Algorithm 2 is in this case very time consuming and very little local search is performed within the time limit. The loop structures seem to be restraining at least for this case and it would be an enhancement to introduce an adaptive layer like the ones seen in ALNS algorithms (Pisinger and Ropke, 2007) to make more frequent and instance specific swaps between the neighborhoods and the local search procedure. Also a rating of the services to be optimized upon could be introduced as the MIP for some services return no moves for implementation in several contiguous for loops.

Since the solution times are generally very low it would be possible to implement a restart functionality with a new initial composition of services with a differing fleet deployment and seeding, perhaps based on the best found solution. Additionally, it is possible to experiment with additional neighborhoods and a more advanced local search procedure, than the one implemented. Additional neighborhoods could target swapping vessel classes between rotations and perform optimization of the port call sequence given the cargo allocated to the service. Speed optimization could also be enhanced. A more advanced local search could be used, where several methods for generating new services could be tested as this is a drawback of the current search especially for the larger instances. Generating routings of very high quality as seen in Brouer et al. (2012) could be implemented as there seems to be time for performing more advanced techniques for generating new promising routings.

5.8.1 Alternative method for generating new services

The local search implemented is relatively simple as described in Section 5.3.7. The services are rated based on the average utilization percentage of all individual voyages on the service. The resulting solution of removing the lowest utilized services is evaluated and at most two services are removed in each iteration. Subsequently new services are added based on the residual demand of the solution with the services removed. The original method creates a single service for each vessel

Instance			Cost(180 days)	Cost(weekly)	imp%	dep%	trans%	Time
Baltic	Low	Best	$3.11 \cdot 10^6$	124419	95.02	100.00	84.83	10.12
		Average	$1.19 \cdot 10^7$	476568	80.24	96.00	79.23	8.57
	Base	Best	$-5.90 \cdot 10^6$	-236038	110.21	100.00	92.07	3.65
		Average	$-7.32 \cdot 10^5$	-29261	101.66	100.00	89.92	4.06
	High	Best	$-5.87 \cdot 10^6$	-234718	107.52	100.00	92.84	9.14
		Average	$1.78 \cdot 10^6$	71144	98.15	96.10	88.21	11.76
WAF	Low	Best	$-9.39 \cdot 10^7$	$-3.76 \cdot 10^6$	221.03	100.00	90.15	13.22
		Average	$-8.24 \cdot 10^7$	$-3.30 \cdot 10^6$	198.82	96.67	84.84	10.89
	Base	Best	$-1.28 \cdot 10^8$	$-5.13 \cdot 10^6$	274.32	90.47	94.88	9.24
		Average	$-1.14 \cdot 10^8$	$-4.56 \cdot 10^6$	254.32	96.43	94.15	8.14
	High	Best	$-1.31 \cdot 10^8$	$-5.25 \cdot 10^6$	412.35	98.04	97.35	13.09
		Average	$-1.02 \cdot 10^8$	$-4.06 \cdot 10^6$	259.79	82.35	89.90	12.63
Mediterranean	Low	Best	$6.05 \cdot 10^7$	$2.42 \cdot 10^6$	42.67	78.26	81.20	155.78
		Average	$6.82 \cdot 10^7$	$2.73 \cdot 10^6$	37.85	86.52	80.98	98.56
	Base	Best	$4.09 \cdot 10^7$	$1.64 \cdot 10^6$	65.65	85.71	91.00	70.37
		Average	$5.36 \cdot 10^7$	$2.14 \cdot 10^6$	49.16	80.00	84.87	77.26
	High	Best	$4.10 \cdot 10^7$	$1.64 \cdot 10^6$	54.18	81.82	94.35	56.57
		Average	$4.78 \cdot 10^7$	$1.91 \cdot 10^6$	59.29	73.94	85.32	91.95
Pacific	Low	Best	$2.10 \cdot 10^8$	$8.40 \cdot 10^6$	70.23	97.56	81.14	359.76
		Average	$2.92 \cdot 10^8$	$1.17 \cdot 10^7$	61.20	92.56	79.29	368.39
	Base	Best	$4.36 \cdot 10^7$	$1.74 \cdot 10^6$	90.65	100.00	93.44	651.85
		Average	$1.62 \cdot 10^8$	$6.47 \cdot 10^6$	78.74	89.00	85.03	512.95
	High	Best	$-4.26 \cdot 10^7$	$-1.71 \cdot 10^6$	109.41	97.48	96.80	833.32
		Average	$1.20 \cdot 10^7$	$4.80 \cdot 10^5$	96.92	90.92	93.20	838.74
WorldSmall	Low	Best	$-4.76 \cdot 10^7$	$-1.90 \cdot 10^6$	101.98	99.02	83.63	5481.76
		Average	$6.93 \cdot 10^7$	2772230	93.09	97.30	79.84	5836.64
	Base	Best	$-7.97 \cdot 10^8$	$-3.19 \cdot 10^7$	178.36	95.82	91.83	8738.23
		Average	$-4.39 \cdot 10^8$	$-1.76 \cdot 10^7$	148.34	93.69	86.44	7881.96
	High	Best	$-1.05 \cdot 10^9$	$-4.18 \cdot 10^7$	433.05	94.64	92.89	4561.85
		Average	$-5.77 \cdot 10^8$	$-2.31 \cdot 10^7$	215.90	95.80	90.09	6735.87
AsiaEurope	Low	Best	$-4.45 \cdot 10^7$	$-1.78 \cdot 10^6$	103.60	100.00	83.17	14405.00
		Average	$7.04 \cdot 10^7$	2815218	94.41	98.14	81.07	13962.74
	Base	Best	$-3.07 \cdot 10^8$	$-1.23 \cdot 10^7$	151.10	98.30	93.18	14426.60
		Average	$-9.22 \cdot 10^7$	$-3.69 \cdot 10^6$	112.17	95.17	86.42	14502.02
	High	Best	$-5.03 \cdot 10^8$	$-2.01 \cdot 10^7$	185.96	96.70	95.70	14569.40
		Average	$-1.32 \cdot 10^8$	5288480	120.18	96.42	90.77	13836.17

Table 5.8: Overview of the results obtained for the benchmark suite *LINER-LIB 2012* using the matheuristic for LSNDP. The tests are based on ten runs with different random seeds. **Instance** is the name of the instance, **Best** is the best solution found, **Average** is the average of all ten runs. **Low** is the low capacitated case, **Base** is the medium capacitated case and **High** is the high capacity case. **Cost(180 days)** is the objective value for a planning horizon of 180 days similar to Brouer et al. (2012), whereas **Cost(weekly)** is the weekly cost/revenue of the network. Note that a negative value represents a profitable network. **imp%** is the percentage improvement from the objective value of the initial solution constructed by the MQKP construction heuristic, **dep%** is the percentage of the fleet deployed in terms of number of vessels, **trans%** is the percentage of demand transported and **Time** is the CPU time in seconds.

class targeting the largest residual demand in terms of the volume multiplied with the revenue per unit. Feasibility of the generated service given the duration of transporting the demand and the available number of vessels in the vessel class is considered in the method. The method is denoted *One Service Single Demand (OSSD)*. The approach creates very crude new services and in particular for the large instances with extensive use of transshipments the new services are

often removed in the subsequent local search as they have not evolved into a promising service. An alternative method (*MQKP*) invokes the construction heuristic for the ports present in the residual demand and the available fleet. We have tested the new method for the *Baltic* and the *WAF* instance. Ten runs of each instance was performed with identical seeds, such that the constructed solutions that are basis for the optimization are identical for both methods. Table 5.9 shows the average and best solutions found by each method. It can be seen that the more advanced method constructing new services using the construction heuristic gives better results for both instances on average. This means that introducing the *MQKP* method for larger instances should be investigated as it might improve the search.

Instance	Method	Best cost (weekly)	Average cost
Baltic	OSSD	-58165	245667
Baltic	MQKP	-41209	58601
WAF	OSSD	-4468130	-3843604
WAF	MQKP	-5128090	-4560574

Table 5.9: Average and Best weekly cost for two different methods of constructing a new service set in the local search. The ten runs are performed with the same random seed such that the basis for the optimization (the constructed solution) is identical for both methods.

5.9 The matheuristic as a decision support tool

The matheuristic can be used as a decision support tool for evaluating and altering a given network. A given network can be used as initial solution and it is furthermore possible to assign a subset of the services for optimization, while maintaining full knowledge of the capacity and flow in the remaining network. The matheuristic will keep the remaining network fixed and only apply the MIP neighborhood and local search to the designated subset of the services, while maintaining full knowledge of the entire network. Network planners are often responsible for a particular trade or geographic area and the method enables optimization of this designated part of the network. The method has been tested on a single case from Maersk Line.

The case is constructed from a real network at Maersk Line. The original network satisfies several constraints that are not incorporated in the matheuristic in its current form, such as several capacity types (DRY, REEFER, High Cube etc.), transit time restrictions, cabotage restrictions, and empty repositioning. Furthermore, the original network uses a larger set of vessel classes than the constructed case. The original network has been modified to fit the input of the matheuristic, which means that the capacity allocation has been set to match 6 vessel classes and demands have been mapped to a single capacity type without transit time restrictions, cabotage rules and empty repositioning. This means that the resulting case will be slightly overcapacitated and fulfills several constraints not considered by the matheuristic. This makes the mapped case an "easy optimization problem" compared to the real life case. Revenues have been constructed as an upper bound of the revenues found in the WorldLarge instance from *LINER-LIB 2012* and therefore, the objective value of the optimization does not represent the real value of the network in any way. Bearing these assumptions in mind, three cases have been constructed based on the mapped case optimizing on 3, 8 and 10 services respectively. These tests were performed on a regular desktop with an *Intel(R) Core(TM)2 Duo CPU P9400* at *2.40GHz* with 4 GB RAM.

The solutions calculated by the matheuristic are encouraging as there is a decent improvement in the cost, which for all cases show a revenue. The improvement seems to be primarily due to removing excess capacity looking at the deployment percentage (**Depl.%**). As a proof-of-concept implementation it shows that the matheuristic is capable of optimizing upon a real sized network with good results. It would be interesting to investigate an implementation fulfilling additional real-life constraints and testing it on a larger variety of cases and scenarios. Possible scenarios are adjustments to changes in demands and/or revenues, alternative fleet deployment and the introduction of new/changed services, that may have effects in other parts of the network.

Case	$ S $	trans%	Depl.%	Cost	Time
Base Case with no optimization	0/139	99.5	100.00	$-5.67 \cdot 10^8$	140.59
Base Case - 3 services optimized	3/139	98.3	98.70	$-5.73 \cdot 10^8$	1862.48
Base Case - 8 services optimized	8/139	99.3	94.48	$-6.09 \cdot 10^8$	2385.66
Base Case - 10 services optimized	10/139	99.3	90.80	$-6.28 \cdot 10^8$	2803.58

Table 5.10: Tests optimizing a partial network based on Maersk Line network. $|S|$ denotes the number of services optimized upon compared to the full number of services in the network. **trans%** is the percentage transported of the total demand in units, **Depl.%** denotes the number of vessels deployed and **Cost** is the objective value (Again we are minimizing and a negative number means a profit). **Time** denotes the CPU time in seconds.

5.10 Conclusions

In this paper a matheuristic for the LSNDP has been presented using the reference model from Brouer et al. (2012) with strict weekly frequencies for all services. The matheuristic consists of a novel construction heuristic viewing the liner shipping network design problem as a specialized variant of the multiple quadratic knapsack problem. The initial solution is then improved using an integer program as neighborhood for each individual service considering insertion and removal of port calls based on estimation functions. The estimation functions allows us to circumvent solving a large scale multicommodity flow problem for evaluating a given candidate solution and may be applied in other heuristics for liner shipping network design. To circumvent the multicommodity flow a delta column generator is used to evaluate a candidate solution. The idea is to reuse the optimal basis for all variables not invalidated by the implemented moves and restart delayed column generation from here. A speedup of a factor 2-10 is achieved on average on the testbed for the *LINER-LIB 2012*. The improvement heuristic is used in combination with a simple local search based on a ruin-and-recreate principle, where vessels are freed by removing low utilized services and the resulting excess vessels are redeployed to services targeting the residual demand of a solution. Computational results are provided for the benchmark suite *LINER-LIB 2012* and are the first results enforcing strict weekly frequency. For twelve out of eighteen instances the algorithm seems to be performing well making a reasonable profit. For two cases (five out of six instances) the solutions are loss giving. Both cases have low or no profit using bi-weekly frequencies in Brouer et al. (2012) indicating that the solution spaces of the test cases do not contain many if any profitable solutions. The pacific case is profitable for bi-weekly frequency as seen in Brouer et al. (2012). The average performance is poor compared to the best solutions found, which indicates that the solution space is complex with many sub-neighborhoods and the matheuristic gets trapped in a local minimum too easily. The heuristic is also highly dependent on the initial solution and it may be worthwhile spending more time creating a good initial network by enhancing the construction heuristic. Finally, the matheuristic has been tested as a decision support tool with an existing liner shipping network as input only optimizing on a subset of the services in the network. Three cases are have been evaluated based on a real-life inspired network. The real life network contains additional constraints on the flow and hence is conceived as suboptimal for the matheuristic as these constraints are not enforced in the optimization. As a results the solutions from the optimized case may not be feasible in a real life network. Nevertheless, results are very encouraging as the algorithm is able to improve on all three cases tested and may prove as a valuable decision support tool for making incremental changes to a network and analyze different scenarios during network configuration.

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Part III

Modelling the Liner shipping Network Design Problem

Chapter 6

A Path Based Model for a Green Liner Shipping Network Design Problem

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Abstract Liner shipping networks are the backbone of international trade providing low transportation cost, which is a major driver of globalization. These networks are under constant pressure to deliver capacity, cost effectiveness and environmentally conscious transport solutions. This article proposes a new path based MIP model for the Liner shipping Network Design Problem minimizing the cost of vessels and their fuel consumption facilitating a green network. The proposed model reduces problem size using a novel aggregation of demands. A decomposition method enabling delayed column generation is presented. The subproblems have similar structure to the subproblems of the Vehicle Routing Problems, which can be solved using dynamic programming. An algorithm has been implemented for this model, unfortunately with discouraging results due to the structure of the subproblem and the lack of proper dominance criteria in the labelling algorithm. Instead we discuss heuristic approaches for solving the pricing problem, and the possibility of solving the model in a heuristic column generation scheme. **Keywords:** liner shipping, network design, mathematical programming, column generation, green logistics

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6.1 Introduction

Global liner shipping companies provide port to port transport of containers, on a network which represents a billion dollar investment in assets and operational costs.

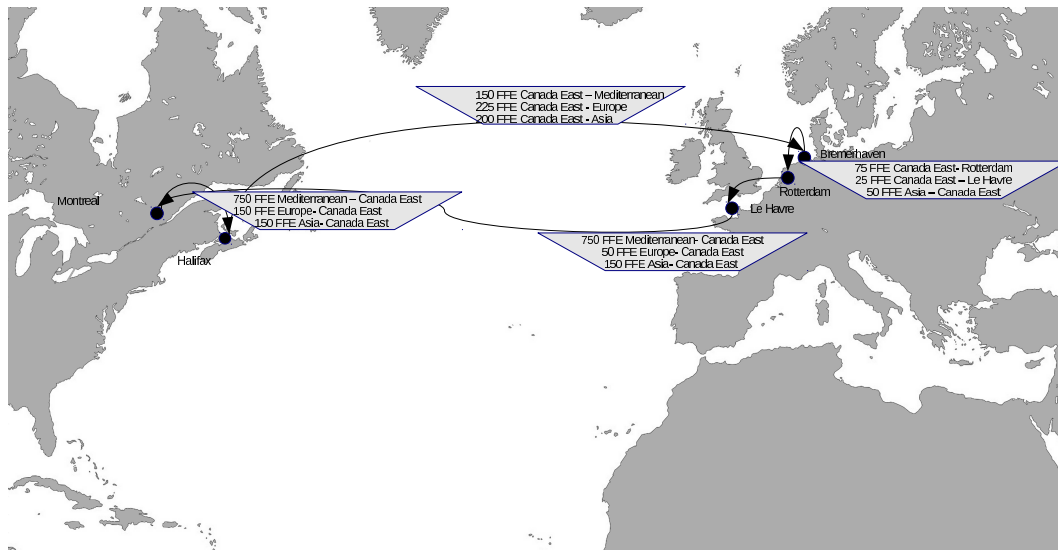


Figure 6.1: A Canada-Northern Europe service. FFE is Forty Foot Equivalent unit container used to express the volume of containers in each cargo category.

The liner shipping network can be viewed as a transportation system for general cargo not unlike an urban mass transit system for commuters, where each route (service) provides transportation links between ports and the ports allow for transshipment in between routes (services). The liner shipping industry is distinct from other maritime transportation modes primarily due to a fixed public schedule with a given frequency of port calls (Stopford, 2009). The network consists of a set of *services*. A *service* connects a sequence of ports in a cycle at a given frequency, usually weekly as an industry standard. In Figure 6.1 a service connecting Montreal-Halifax and Europe is illustrated. The weekly frequency means that several vessels are committed to the service as illustrated in the figure, where four vessels cover a round trip of 28 days placed with one week in between vessels. This round trip for the vessel is referred to as a *rotation*. Note that the Montreal service carries cargo to North Europe, the Mediterranean and Asia, with the two latter transshipping in Bremerhaven. In a similar way cargo headed for Canada has multiple origins. This illustrates that transshipments to other connecting services is at the core of liner shipping. Therefore, the design of a service is complex, as the set of rotations and their interaction through transshipment is a transportation system extending the supply chains of a multiplicity of businesses. Figure 6.2 illustrates two services interacting in transporting goods between Montreal-Halifax and the Mediterranean, while individually securing transport between Montreal-Halifax and Northern Europe, and Northern Europe and the Mediterranean respectively. The Montreal service additionally interacts with a service between Europe and Asia, which is partly illustrated.

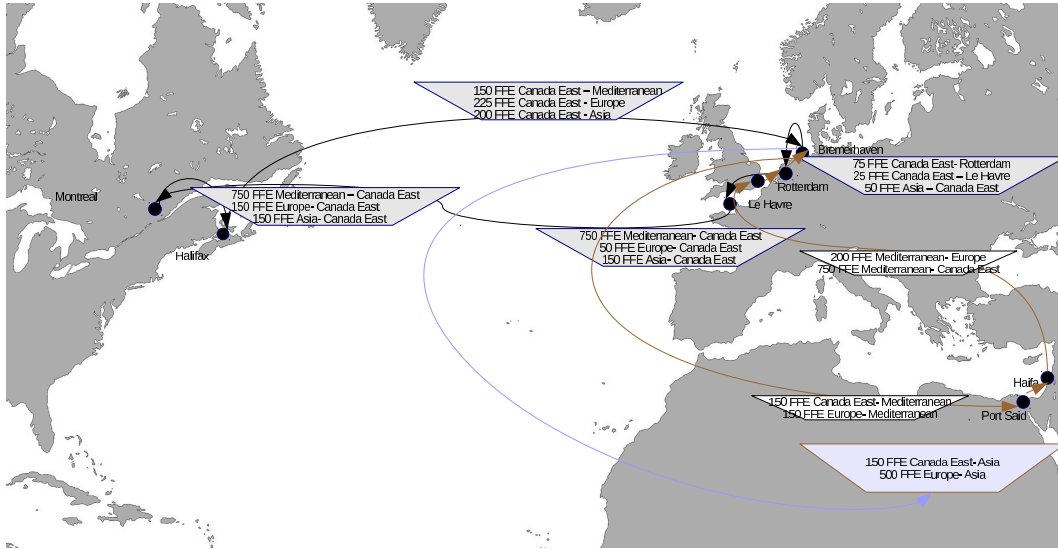


Figure 6.2: Two connecting services. The Montreal service from Figure 6.1 and a Europe-Mediterranean service with a round trip time of 2 weeks illustrated by two white vessels. The cargo composition on board vessels illustrate transshipments at the core of the liner shipping network design. The light blue incomplete service illustrates a larger service transporting cargo between Europe and Asia.

6.1.1 Modelling the Liner Shipping Network Design Problem (LSNDP)

The Liner Shipping Network Design Problem (LSNDP) aims to optimize the design of the networks to minimize cost, while satisfying customer service requirements and operational constraints. The mathematical formulation of the LSNDP may be very rich as seen in Løfstedt et al. (2011), where a compact formulation along with an extensive set of service requirements and network restrictions is presented. A rich formulation like the one presented in Løfstedt et al. (2011) serves as a description of the LSNDP domain, but is not computationally tractable as the number of feasible services is exponential in the number of ports. Therefore, a formulation of the LSNDP is typically restricted to an interpretation of the domain along with the core costs and constraint structures of the problem. The LSNDP has been modelled as a rich Vehicle Routing Problem (VRP) (Baldacci et al., 2010), where transshipments are not allowed and vessels can be assumed to return empty to a single main port of a voyage in, e.g., Fagerholt (2004) and Karlaftis et al. (2009). The structure is applicable for regional liner shippers referred to as *feeder* services as opposed to global liner shipping in focus in the present paper. Models where the LSNDP is considered as a specialized capacitated network design problem with multiple commodities are found in Reinhardt and Kallehauge (2007), Agarwal and Ergun (2008), Alvarez (2009), and Plum (2010). The network design problem is complicated by the network consisting of disjoint cycles representing container vessel routes as opposed to individual links. The models allow for transshipments, but transshipment cost is not always part of the objective (e.g., Agarwal and Ergun (2008)). The vessels are not required to be empty at any time. The works of Agarwal and Ergun (2008); Alvarez (2009) identify a two tier structure of constraint blocks: the first deciding the rotations of a single or a collection of vessels resulting in a capacitated network and the second regarding a standard multicommodity

flow problem with a dense commodity matrix. The cost structure of LSNDP places vessel related costs in the first tier and cargo handling cost and revenue in the second tier. The work of Plum (2010) has identified two main issues with solving the LSNDP as a specialized capacitated network design problem:

1. Economy of scale on vessels and the division of cost and revenue on the two tiers results in highly fractional LP solutions.
2. The degeneracy of the multicommodity flow problem results in weak LP bounds.

Furthermore, it is well known that the linear multicommodity flow problem and hence capacitated network design problems are increasingly complex to solve with the number of distinct commodities. Computational results for existing models confirm the hardness of this problem and the scalability issues, struggling to solve instances with 10-15 ports and 50-100 commodities.

The model presented in this paper has a single tier and combines revenue with total cost in the service generation problem. The motivation is to ensure efficient capacity utilization of vessels and avoid highly fractional LP solutions. Service generation is based on pick-up-and-delivery of cargoes transported entirely or partly on the service. The cost of a service reflects asset, operational and port call costs of the vessels on the service, along with the cargo handling cost and revenue of collected cargo on the service. The cargo handling cost includes load, unload and transshipment costs. The model is inspired by the Pick-up-and-Delivery VRP problem, but is considerably more complex as we allow transshipments on non-simple cyclic routes, where the vessel is not required to be empty at any point in time.

The degeneracy of the multicommodity flow problem is mitigated both by modeling the flow as assignments to services as opposed to the traditional multicommodity flow formulation, but also by exploiting the liner shipping concept of *trade lanes* to aggregate the number of distinct commodities to a minimum. Trade lanes are based on the geographic distances within a set of ports and their potential to import/export to another region.

Maritime shipping produces an estimated 2.7% of the world's CO_2 emission, whereof 25% is accounted to container vessels according to the Council (2010). Many liner shipping companies focus on the environmental impact of their operation and the concept of *slow steaming* has become a value proposition for some liner shipping companies List (2010). Cariou (2011) estimate that the emissions have decreased by 11 % since 2008 by slow steaming alone. A break down of the cost of a service to each vessel Stopford (2009) state that 35-50% of the cost is for fuel (bunker) whereas capital cost accounts for 30-45%, OPEX (crew, maintenance and insurance) accounts for 6-17% and port cost for 9-14%. Slow steaming minimizes the fuel cost, but comes at an asset cost of additional vessels deployed to maintain weekly frequency (Notteboom and Vernimmen, 2009). Slow steaming is not always an option as some cargo may have crucial transit times. Current models of LSNDP assumes fixed speed on a service. The model of Alvarez (2009) explicitly aims at minimizing the fuel cost and consumption in the network by varying the speed of services in the model. The works of Løfstedt et al. (2011); Notteboom and Vernimmen (2009); Fagerholt et al. (2009) state that the speed on a service is variable on each individual voyage between two ports. Calculating fuel consumption based on an average fixed speed on a roundtrip is an approximation, as the fuel consumption is a cubic function of speed (Stopford, 2009). As a result the actual fuel consumption of a service cannot be estimated until the schedule is fixed. Tramp shipping companies often model their routing and scheduling problem as rich Pick-up-and-Delivery VRP problems with Time Windows (Fagerholt and Lindstad, 2007; J. E. Korsvik and Laporte, 2011). Fagerholt et al. (2009) is the first article within tramp shipping with variable speed between each port pair in the routing. The optimization of speed and hence minimizing the fuel consumption and environmental impact is driven by the time windows and the optional revenue of spot cargoes. (Fagerholt et al., 2009; I. Norstad and Laporte, 2011) report significant improvements in solution cost using variable speed. Minimizing the fuel consumption of the network can be a post optimization regarding speed of the liner shipping network, when deciding on the schedule in terms of berthing windows or the transit time of individual cargo routings. The path based model presented in this paper assumes a fixed speed for each vessel class and in the

dynamic programming algorithm the number of vessels deployed to a service is rounded up to the nearest integer in order to ensure that a weekly frequency can be maintained on each service.

The path based model is inspired by operations research techniques within the airline industry, where the optimization is divided into faces. Therefore, a solution to the path based model is a generic capacitated network of cyclic services based on a weekly frequency of port calls. The generic network is transformed into an actual network by deciding a specific schedule, deploying vessels and deciding on the speed of the individual voyages and actual flow of all distinct commodities. The slow steaming speed of a vessel is 12 knots and depending on size and age a vessel has a maximal speed of 18 to 25 knots. If the fixed speed is chosen 30-40% above slow steaming speed for each vessel class, rounding up the number of vessels will allow post optimization of the schedule to achieve an energy efficient network with focus on slow steaming, while ensuring the transit time of products. The generic network facilitates the design of a green liner shipping network, while at the same time enabling scalability due to a more general description of the network.

6.1.2 Demand Aggregation

In models of the LSNBP using a specialized capacitated network design formulation the second tier is a standard multicommodity flow problem. The work of Alvarez (2009) identifies solving the multicommodity flow problem as prohibitive for larger problem instances due to the large number of commodities considered. In Alvarez (2009) the commodities are aggregated by destination, giving a smaller model to solve. This could result in worse LP bounds as identified in Croxton et al. (2007), since the LSNBP will have a concave cost function, due to the economies of scales of deploying larger vessels, and high start up costs, as at least one vessel must be deployed.

A contribution of this paper is to formulate a model that considers aggregated aspects of the demand instead of specific origin-destination ($o-d$) pairs. This is motivated by the *trade*-centric view of liner-shipping present in the liner shipping industry instead of the $o-d$ -centric view considered in the literature. As seen in Figure 6.1, the ($o-d$ demand from Halifax to Rotterdam could be considered, but in practice it will be hard to estimate such a specific demand. More realistically one could estimate the volume of exports from Halifax to Northern Europe and reversely the volume of imports from East Coast Canada to Rotterdam (or exports from Mediterranean to Halifax as in Figure 6.2). Each commodity $k \in K$ will then be characterized by a volume d^{XY} from a region X to a region Y i.e. East Coast Canada or Northern Europe as seen in Figure 6.1 on the vessels in deep sea. Each set of X, Y will symbolize a *trade*. Each port $p \in X$ will also have an export and import in the trade: d^{pY}, d^{Xp} , where $\sum_{p \in X} d^{pY} = \sum_{p \in Y} d^{Xp}$ as seen in Figure 6.1 on the vessels in a region. In effect a port as Halifax will be ensured a volume of export to Mediterranean ports and each of these will be insured a volume of imports from East Coast Canadian ports, without specifying the concrete origin-destination pairs. Note the difference in aggregation approach, compared with the models of Croxton et al. (2007), as we are now aggregating by *trade* origin-region to destination-region, instead of aggregation by destination port. This should give the benefit of fewer variables due to the aggregation, while we still have quite tight LP-relaxations.

The aggregation of demand may be more or less fine grained according to the definition of ports, regions and trade lanes, enabling both detailed networks for a smaller region and coarse network designs for a larger set of ports that may be refined by subsequent optimization methods. We foresee a computational tractability trade-off between the number of ports and the number of distinct commodities when defining regions for ports.

This can also be seen in the light of forecasting accuracy, usually the more detailed the level of forecasting is the more inaccurate it will be. This allows a forecasting to be done at a more natural level, i.e. on total trade volumes and total port import and export volumes.

In the following we will present a path-based formulation of the LSNBP and a column generation approach generating capacitated, cyclic rotations with assigned flow. We will outline a dynamic programming algorithm to solve the pricing problem. Preliminary computational results of an implementation of the algorithm will be given, which reveals poor performance for solving the pricing problem. This leads us to believe that alternative methods must be developed to efficiently solve the pricing problem, for the approach to be able to solve instances of a significant size. This

work is an extension of a contribution to the proceedings of IMECS 2011 of Jepsen, Løfstedt, Plum, Pisinger, and Sigurd (2011).

6.2 Service Based Model

In the following we introduce a model based on a combination of feasible services for each vessel class, into a generic liner shipping network solution. The service based model is based on a Dantzig-Wolfe decomposition of the model presented in Løfstedt et al. (2011). Let S_v denote the set of feasible services for a vessel class $v \in V$ and let $S = \cup_{v \in V} S_v$. Let α_{kps}^{XY} and β_{kps}^{XY} be the amount of respectively load and unload of containers from region X to region Y on the k 'th visit to port p on service $s \in S$. We assume that $\alpha_{kps}^{XY} = \beta_{kps}^{XY} = 0, \forall p \notin X \cup Y \cup G^{XY}$, where G^{XY} is the set of ports where transshipments is allowed for trade XY . Let M_p be the maximal number of port visits to port p for each service. Furthermore, let γ_{pq} equal the number of times the service sails between ports $p \in P$ and $q \in P$. The move cost in a port p for a trade $XY \in K$ consist of the unload cost u_p^{XY} and load cost l_p^{XY} . For ports $p \in X$ the transshipment cost is included in the unload cost and the revenue is r_p^{XY} . For ports $p \in P \setminus X$ the transshipment cost is included in the load cost. Each vessel of vessel type $v \in V$ has costs c_v for fuel-, crew- and depreciation of vessel value or time-charter-costs per week. The cost of vessel type v calling a port q is c_q^v . The number of vessels used by the service is the round trip distance of the service divided by W_d^v , the weekly distance covered by vessel type v at the predefined speed. This value is rounded up to ensure the vessels can complete the round trip at the predefined speed. The number of vessels used by the service is given as $n_s = \left\lceil \sum_{p \in P} \sum_{q \in P} \frac{d_{pq} \gamma_{pqs}}{W_d^v} \right\rceil$. The cost of a service $s \in S$ is given as:

$$\begin{aligned} c_s = & \sum_{XY \in K} \sum_{p \in X} \sum_{k \in M_p} r_p^{XY} (\alpha_{kps}^{XY} - \beta_{kps}^{XY}) \\ & - \sum_{XY \in K} \sum_{p \in P} \sum_{k \in M_p} (l_p^{XY} \alpha_{kps}^{XY} + u_p^{XY} \beta_{kps}^{XY}) \\ & - c_v n_s - \sum_{p \in P} \sum_{q \in P} c_q^v \gamma_{pqs} \end{aligned}$$

The model based on services is as follows:

$$\max \sum_{s \in S} c_s \lambda_s \quad (6.1)$$

$$\text{s.t.} \quad 0 \leq \sum_{s \in S} \sum_{k \in M_p} (\alpha_{pks}^{XY} - \beta_{pks}^{XY}) \lambda_s \leq d^{pY} \quad \forall XY \in K, \forall p \in X \quad (6.2)$$

$$0 \geq \sum_{s \in S} \sum_{k \in M_p} (\alpha_{pks}^{XY} - \beta_{pks}^{XY}) \lambda_s \geq -d^{Xp} \quad \forall XY \in K, \forall p \in Y \quad (6.3)$$

$$\sum_{s \in S} \sum_{k \in M_p} (\alpha_{pks}^{XY} - \beta_{pks}^{XY}) \lambda_s = 0 \quad \forall p \in G^{XY}, \forall XY \in K \quad (6.4)$$

$$\sum_{s \in S} \sum_{p \in X \cup Y} \sum_{k \in M_p} (\alpha_{pks}^{XY} - \beta_{pks}^{XY}) \lambda_s = 0 \quad \forall XY \in K \quad (6.5)$$

$$\sum_{s \in S_v} n_s \lambda_s \leq |v| \quad \forall v \in V \quad (6.6)$$

$$\alpha_{kps}^{XY}, \beta_{kps}^{XY} \in \mathbb{Z}^+ \quad \forall s \in S, \forall XY, \forall p \in X, \forall k \in M_p \quad (6.7)$$

$$\lambda_s \in \{0, 1\} \quad \forall s \in S \quad (6.8)$$

The objective (6.1) maximizes the profit, constraints (6.2) and (6.3) ensure that the difference between what is loaded and unloaded (unloaded and loaded) by all services in a port is positive

and less than the export capacity (import capacity) of the port for the given *trade*. Constraints (6.4) ensure that the amount of containers loaded equals the amount of containers unloaded in a transshipment port and constraints (6.5) ensure that all containers loaded are unloaded for each *trade*. Constraints (6.6) ensure that the number of available vessels for each vessel class is not exceeded and the binary domain on the variables is defined by (6.8).

The key issue with the service based model is that the set of feasible services S can be exponential in the number of ports. Therefore, we cannot expect to solve instances of significant size. To overcome this issue we propose to write up the model gradually using delayed column generation and then solve the problem through Branch-and-Cut-and-Price. Branching is done by imposing a limit on the number of times an arc can be used by a given vessel class. We will investigate the possibility of applying an enumeration technique similar to the one used within CVRP (Baldacci et al., 2008).

6.2.1 Pricing Problem

The pricing problem calculates a non-simple cycle σ centered around any starting node p_s with associated loads and unloads. The cycle respects the capacity of the vessel class, C_v , at every port p , ensures feasibility of a weekly frequency for the vessel class v given the distance of the schedule, and lastly, that port p is visited no more than M_p times $\forall p \in P$. The pricing problem returns a variable representing a load and an unload pattern, which implicitly defines a non-simple cycle starting and ending at the same port $p \in P^v$, deploying n_s vessels to maintain weekly frequency at the fixed speed enforced on the service pattern. The above problem has a similar structure to the pricing problems of Vehicle Routing Problems modelled as a Resource Constrained Shortest Path problem (see Irnich and Desaulniers (2005)). The Resource Constrained shortest Path Problem is often solved by label setting algorithms. As it is possible for the demand to be split on different paths, we need to ensure that we allow all possibilities of transshipments. This necessitates that, labels are created for each integral unit of the demand up to the minimum of the available capacity or the demand.

Objective function of the pricing problem

The objective function of the pricing problem is to find the best reduced cost of a master problem variable at the given iteration of the master problem. For each $XY \in K$ a port $p \in P$ is present in at most one of the constraints (6.2) to (6.4). Let $\omega_p^{XY}, \forall XY \in K, \forall p \in X \cup Y \cup G^{XY}$ denote the duals from (6.2) to (6.4). Let δ^{XY} be the dual variables of constraints (6.5) and π^v are the duals of constraints (6.6).

For each vessel class $v \in V$ the reduced cost of a service (column) $s \in S_v$

$$\begin{aligned} \hat{c}_s = & c_s - \sum_{XY \in K} \sum_{p \in X \cup Y \cup G^{XY}} \sum_{k \in M_p} \omega_p^{XY} (\alpha_{kps}^{XY} - \beta_{kps}^{XY}) \\ & - \sum_{XY \in K} \sum_{p \in X \cup Y} \sum_{k \in M_p} \delta^{XY} (\alpha_{kps}^{XY} - \beta_{kps}^{XY}) - \pi_v n_s \end{aligned}$$

Expanding the term \hat{c}_s and rearranging the terms according to load and unload combined with the port belonging to either X, Y or G_k we obtain the following reduced cost:

$$\begin{aligned}
\hat{c}_s = & \sum_{XY \in K} \sum_{p \in X} \sum_{k \in M_p} (r_p^{XY} - l_p^{XY} - \omega_p^{XY} - \delta^{XY}) \alpha_{kps}^{XY} \\
& + \sum_{XY \in K} \sum_{p \in X} \sum_{k \in M_p} (-r_p^{XY} - u_p^{XY} + \omega_p^{XY} + \delta^{XY}) \beta_{kps}^{XY} \\
& + \sum_{XY \in K} \sum_{p \in Y} \sum_{k \in M_p} (-l_p^{XY} - \omega_p^{XY} - \delta^{XY}) \alpha_{kps}^{XY} \\
& + \sum_{XY \in K} \sum_{p \in Y} \sum_{k \in M_p} (-u_p^{XY} + \omega_p^{XY} + \delta^{XY}) \beta_{kps}^{XY} \\
& + \sum_{XY \in K} \sum_{p \in G^{XY}} \sum_{k \in M_p} (-l_p^{XY} - \omega_p^{XY}) \alpha_{kps}^{XY} \\
& + \sum_{XY \in K} \sum_{p \in G^{XY}} \sum_{k \in M_p} (-u_p^{XY} + \omega_p^{XY}) \beta_{kps}^{XY} \\
& - (\pi_v + c_v) n_s - \sum_{p \in P} \sum_{q \in P} c_q^v \gamma_{pq}
\end{aligned}$$

The reduced cost can be rewritten as a cost connected to loading, unloading, and sailing in terms of the number of vessels deployed and the cumulative port call cost. The cost of (un)loading a demand from trade XY depends on the region of the port. If the port is from the origin region X a revenue is obtained for loading and subtracted for unloading at the port. This ensures that revenue is only collected at the initial load. The costs are the (un)load cost, and the dual values from constraints (6.2)-(6.4) concerning the flow conservation and the dual value from the flow balance constraint for the trade (6.5). If the port is from the destination region Y the cost is the (un)load cost, and the dual values from constraints (6.2)-(6.4) concerning the flow conservation and the dual value from (6.5). For a transshipment port $p \in G^{XY}$ the cost is only related to (un)load cost and the dual values of (6.2)-(6.4).

$$\begin{aligned}
\hat{l}_p^{XY} &= \begin{cases} r_p^{XY} - l_p^{XY} - \omega_p^{XY} - \delta^{XY} & \forall p \in X \\ -l_p^{XY} - \omega_p^{XY} - \delta^{XY} & \forall p \in Y \\ -l_p^{XY} - \omega_p^{XY} & \forall p \in G^{XY} \end{cases} \\
\hat{u}_p^{XY} &= \begin{cases} -r_p^{XY} - u_p^{XY} + \omega_p^{XY} + \delta^{XY} & \forall p \in X \\ -u_p^{XY} + \omega_p^{XY} + \delta^{XY} & \forall p \in Y \\ -u_p^{XY} + \omega_p^{XY} & \forall p \in G^{XY} \end{cases}
\end{aligned}$$

Finally, the port call cost c_q^v is paid upon each sailing/extension onto a new port $p \in P$ and the cost $\hat{c}_v = \pi_v + c_v$ is inferred each time the distance of W_d^v is traveled.

Label setting algorithm for LSNDP

The $|V|$ pricing problems for each vessel class can be formulated as the following graph problem. Given a directed graph $G^v = (N^v, A^v)$ where the node set is $N^v = P^v \cup L^v \cup U^v$. P^v is the set of ports $\in P$ compatible with vessel class v , $L^v = \bigcup_{w \in P^v} L_w$ the set of load nodes. The sets $L_w = \{\rho_w^{XY} | \forall XY \in K, w \in X \vee Y \vee G^{XY}\}$ represents all possible loads at port w , $U^v = \bigcup_{w \in P^v} U_w$ is the set of unload nodes. The sets $U_w = \{\mu_w^{XY} | \forall XY \in K, w \in X \vee Y \vee G^{XY}\}$ represents all possible unloads at port w . In order to correctly identify transshipments and unloads of a *trade* each demand $XY \in K$ is associated with a set of load nodes $L^{XY} \subseteq L^v$ and a set of unload nodes $U^{XY} \subseteq U^v$, where $L^{XY} = \{\rho_w^{XY} | w \in X \cup Y \cup G^{XY}\}$ and $U^{XY} = \{\mu_w^{XY} | w \in X \cup Y \cup G^{XY}\}$.

The arc set is $A^v = A_s \cup A_u \cup A_l$. Define the function $h : U^v \cup L^v \rightarrow P^v, L_q \mapsto q, U_q \mapsto q$ for mapping between the load and unload nodes and the actual port $q \in P^v$ of the (un)load. The set of sailing arcs is defined as follows $A_s = \{(i, j) | i \in L^v \cup U^v, j \in P^v \setminus \{h(i)\}\}$, the set

of unload arcs $A_u = \{(i, j) | i \in P^v, j \in U_i\} \cup \{(i, j) | i \in U^v, j \in U_{h(i)}\}$ and the set of load arcs $A_l = \{(i, j) | i \in P^v, j \in L_i\} \cup \{(i, j) | i \in U^v, j \in L_{h(i)} \setminus \{\rho_{h(i)}^{XY}\}\} \cup \{(i, j) | i \in L^v, j \in L_{h(i)}\}$. The graph topology is illustrated in Figure 6.3. The distance of an arc depends on the arc type:

$$d_{ij} = \begin{cases} d_{h(i)j} & (i, j) \in A_s \\ 0 & (i, j) \in A_l \cup A_u \end{cases} \quad (6.9)$$

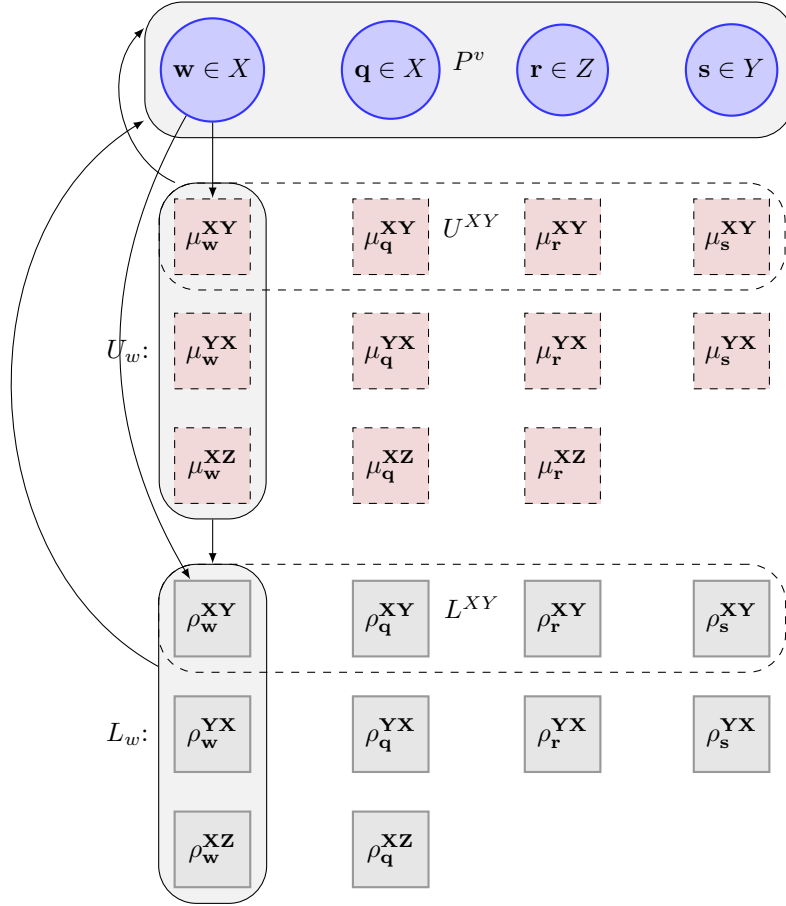


Figure 6.3: A network representation of a graph associated with the label setting algorithm. The set of port call nodes P^v (blue nodes) form a clique. For port $w \in P^v$ the sets U_w (light red nodes), L_w (grey nodes) are illustrated. They represent possible loads and unloads at port w . The sets U_w, L_w form a cliques. A path in the network will follow sequences of $n \in P_v \rightarrow U_n \rightarrow L_n \rightarrow m \in P_v$. It is possible to only unload or load. The load set of a port w is not connected to the unload set of w . Each trade $XY \in K$ is associated with a loadset L^{XY} and an unloadset U^{XY} as illustrated.

In a label setting algorithm, a label E_i is associated with a node i and represents a (partial) path with a (reduced) cost C of the service and a number of resources θ accumulated along the path. A resource may be associated with lower and upper bounds often referred to as a *resource window*. The proposed pricing problem differs significantly from the Elementary Shortest Path Problem with Resource Constraints (ESPPRC) known from VRP:

- The path is not elementary as $M_p \geq 1$.
- The path represents a cycle, σ .
- It is a longest cycle problem as the reduced cost $\hat{c}_s \geq 0$.

- We do not have a designated starting node and hence will have to start the algorithm in every possible port $p \in P^v$.
- The ability to perform a load on the partial path, which can be unloaded at a previous node of the cycle σ . A second pass of all ports in the cycle σ must be performed only allowing the unload extension function to check for load balance.
- There are multiple commodities.
- The route is combined with a loading/unloading pattern not unlike the labelling algorithm for the SDVRPTW in Desaulniers (2010).

In the label setting algorithm for LSNP a label E contains the following information:

- Current port, p_c
- Start port, p_s
- (reduced) cost, t
- Accumulated distance, d_s
- The load of each trade, $F^{XY} \quad \forall XY \in K$
- Current load, $F_c = \sum_{XY \in K} F^{XY}$
- Visit number, $k_p \quad \forall p \in P^v$

The resources are $d_s, (F^{XY})_{XY \in K}, F_c, (k_p)_{p \in P^v}$ i.e. we have $2 + |K| + |P^v|$ resources. The extension function (Irnich and Desaulniers, 2005) of the distance is defined as $e_{(ij)}^d(E_i) = d(E_i) + d_{ij}$. The feasibility and resource consumption of extending label E_i along an arc depends on the arc type:

- *Case 1: extending along a sail arc $(i, j) \in A_s$*

A feasible extension of label E_i to node j along a sail arc $(i, j) \in A_s$ must satisfy the following conditions:

$$\left\lceil \frac{e_{(ij)}^d(E_i)}{W_d^v} \right\rceil \leq |v| \quad (6.10)$$

$$k_j^i + 1 \leq M_j \quad (6.11)$$

Here, (6.10) ensures the feasibility of the number of vessels deployed to the service and (6.11) ensures the number of port calls to port j does not exceed M_j . If the extension is feasible a new label E_j is created. Define

$$\varpi = \left\lceil \frac{e_{(ij)}^d(E_i)}{W_d^v} \right\rceil - \left\lceil \frac{d(E_i)}{W_d^v} \right\rceil \quad (6.12)$$

ϖ expresses whether the label extension will require an additional vessel on the service to maintain weekly frequency. The following extension functions are applied to create label E_j : $p_c^j = j, p_s^j = p_s^i, t^j = t^i - c_j^v - \hat{c}_v \cdot \varpi, d = e_{(ij)}^d(E_i), F_C^j = F_C^i, F_j^{XY} = F_i^{XY}, k_j^j = k_j^i + 1, k_p^j = k_p^i \quad \forall p \in P^v \setminus \{j\}$

- *Case 2: extending along an unload arc $(i, j) \in A_u, j = \mu_p^{XY}$*

A feasible extension of label E_i to node j along unload arc $(i, j) \in A_u$ must satisfy the following conditions:

$$F^{XY} > 0 \quad (6.13)$$

where (6.13) ensures that the commodity XY is currently loaded on the vessel i.e. that a previous visit to a node in L^{XY} has been performed. To ensure that all possible transshipment and unload patterns are considered all integral unloads in $o \in \{1, \dots, \max\{d^{XY}, F_i^{XY}\}\}$ are created with separate labels.

If the extension is feasible a new label E_j^o is created using the extension functions:

$$p_c^j = h(j), p_s^j = p_s^i, t^j = t^i + \hat{u}_p^{XY} \cdot o, d = e_{(ij)}^d(E_i), F_C^j = F_C^i - o, F_j^{XY} = F_i^{XY} - o, F_j^{ZW} = F_i^{ZW} \forall ZW \in K \setminus \{XY\}, k_p^j = k_p^i \quad \forall p \in P^v.$$

- *Case 3: extending along a load arc* $(i, j) \in A_l, j = \rho_p^{XY}$

A feasible extension of label E_i to node j along a load arc $(i, j) \in A_l$ must satisfy the following conditions:

$$F_c^i < C_v \quad (6.14)$$

(6.14) ensures that the vessel has excess capacity for loading. To ensure that all possible transshipment and load patterns are considered all integral loads in $o \in \{1, \dots, \max\{d^{XY}, C_v - F_C^i\}\}$ are created with separate labels. If the extension is feasible a new label E_j^o is created with the following extension function: $p_c^j = h(j), p_s^j = p_s^i, t^j = t^i + \hat{l} \cdot o, d = e_{(ij)}^d(E_i), F_C^j = F_C^i + o, F_j^{XY} = F_i^{XY} + o, F_j^{ZW} = F_i^{ZW} \forall ZW \in K \setminus \{XY\}, k_p^j = k_p^i \quad \forall p \in P^v.$

A state is feasible when the start node is reached ($p_c = p_s$) and the containers are balanced for all trades ($F^{XY} = 0 \quad \forall XY \in K$) by applying unload extensions to the cycle starting from p_s ending in p_s . To obtain the solution to a service the auxiliary data of what has actually been loaded and unloaded has to be stored and a mapping from L to α and from U to β creates the column entries for (un)load in the master problem. For an exact solution to the pricing problem the service with the best reduced cost ($\max \hat{c}_s$) is added to the master problem. However, the label setting algorithm may find several services, where the cost t is greater than 0 and add several columns in an iteration to accelerate convergence of the column generation algorithm.

6.2.2 Dominance

In order to dominate a label it must hold that the dominating label has the same possibilities for extensions and that no extension of the dominated label can yield a better reduced cost than the dominating label.

A label E_1 dominates a label E_2 if the following holds

- $p_c^1 = p_c^2$
- $p_s^1 = p_s^2$
- $t_1 \geq t_2$
- $d_s^1 \leq d_s^2$
- $k_p^1 \leq k_p^2 \quad \forall p \in P^v$
- $F_c^1 \leq F_c^2$
- $F_1^{XY} = F_2^{XY} \quad \forall XY \in K$

Requiring the cargo loads to be identical gives rise to a weak dominance criteria. This means that the labelling algorithm resorts to being practically brute force and a vast number of labels are generated even for relatively small instances. In recent work on dominance criteria for the Pick-up-and-Delivery problem (Ropke and Cordeau, 2009) the dominance criteria for the cargo loads are strengthened by relaxing such that in our case we would have

- $F_1^{XY} \leq F_2^{XY} \quad \forall XY \in K$

if the delivery triangle inequality defined by Ropke and Cordeau (2009) as $d_{ij} + d_{jk} \geq d_{ik}$ holds $\forall i, j, k \in V$. Here j is a delivery node. It is however not trivial to see, whether this relaxation holds for the pricing problem in this paper as each commodity may have several delivery nodes attached and there are no precedence relation between pickup and delivery nodes, due to the cyclic nature of a route.

6.2.3 Complexity

Let T denote an upper bound on the distance of a service. The running time of the label setting algorithm can be shown to be

$O((T|P||C|^{|K|} \prod_{p \in X} d^{p^Y} \prod_{p \in G^{XY}} C)^2)$. Increasing the number of *trades* and the number of transshipment ports will increase the number of states in the Dynamic Programming algorithm. To solve practical problem instances it is therefore important to make a careful choice of the *trades* and the ports, where transshipment is allowed.

6.2.4 Relaxation of pricing problem

In CVRP a pseudo polynomial relaxation is used when solving the strongly NP-hard pricing problem (Baldacci et al., 2008) to reduce the practical running time of the algorithm. The method has proven to be very powerful for the CVRP. A pseudo polynomial relaxation of our pricing problem can be obtained as follows: Each port is assigned the minimal load and unload cost and the bounds on the load are removed. In each port the number of different states will then be limited to $T|P||C|$ and a running time of $O(T|P|^2|C|)$ can be obtained. However, defining a strong bound for the minimal load and unload cost for each port is not trivial as several commodities may origin or transship at a given node and further research must be conducted in order to achieve a relaxation with a good bound.

As the pricing problem is very complex, we need not solve the pricing problem to optimality in each iteration, but one could stop once a sufficient amount of columns with positive reduced cost has been found. An easy way to do this is to run the dynamic programming algorithm using a greedy variant adding any reduced cost column instead of the best reduced cost column.

6.3 Preliminary computational Results

The described algorithm has been implemented using CPLEX to solve the master problem and a labelling algorithm to solve the pricing problem. The results are currently not satisfactory for solving the pricing problem. The structure of the labelling algorithm, the lack of proper dominance criteria and especially the need to generate labels for all integral steps of load and unloads ($o \in \{1, \dots, \max\{d^{XY}, F_i^{XY}\}\}$ respectively $o \in \{1, \dots, \max\{d^{XY}, C_v - F_C^i\}\}$) creates a huge number of very similar labels. The combinations of these causes the labelling algorithm to effectively be a brute force algorithm in an extremely large search space. Even for very small graphs ($n = 4$) the number of considered labels are in the 10'ths of millions.

A simplification of the model is to only consider demand paths, which are fully loaded or unloaded either regarding the demand or capacity, i.e. $o = \{F_i^{XY}, d^{XY}\}$ respectively $o = \max\{d^{XY}, C_v - F_C^i\}$. Unfortunately, this approach is inconsistent with the idea of aggregated demands, as these *will* need to split to reach their respective origins / destinations, discouraging this direction.

As a result we have not pursued methods such as bounding to improve upon the current algorithm of the pricing problem as we believe alternative solution methods must be applied for an efficient algorithm to solve the pricing problem. Another alternative is the design and implementation of efficient heuristics to generate variables and subsequently solve a heuristic implementation of a Branch-and-Price algorithm similar to the one seen in Agarwal and Ergun (2008).

6.4 Conclusion

We have presented a new model for LSNDP. Among the benefits of the proposed model is a novel view of demands in liner-shipping, which are considered on a *trade* basis. This has the advantage of giving a natural understanding, and requiring fewer variables. The model assigns cargo to routes, which may result in a tighter search space for a branch-and-bound algorithm.

A solution approach using delayed column generation has been presented, where the proposed subproblem is related to the pricing problems in VRP, where Branch-&-Cut-&-Price has been used with great success. We have discussed a pseudo polynomial relaxation to be used as bounding function, when solving the pricing problem in combination with heuristics and other techniques that have been effective in solving VRP problems. In the VRP context resource limitations have proven to be effective for the dynamic programming algorithms in reducing the state space. In the dynamic programming algorithm presented in this paper these resource limitations do not reduce complexity of the subproblem sufficiently, because dominance criterions are different. The proposed algorithm has been implemented but showed disappointing results, due to the lack of dominance criteria and a large search space for the label setting algorithm. We still believe that the main ideas in this paper can be useful to solve the LSNDP, i.e. the thoughts of combining cost and revenue in a single pricing problem and especially the notion of demand aggregation, which lends to a natural understanding in Liner Shipping. However, we must conclude that alternative methods or extensions of the current dynamic programming algorithm will be needed to solve a pricing problem, where cargo load patterns for multiple commodities are combined with a routing.

Further work with richer formulations of LSNDP, considering aspects as transit time limits on paths, and other operational constraints from liner shipping will tighten the search space of the pricing problems. However, it is uncertain whether additional real-life complexity in the pricing problem will allow for effective dominance criteria in a label setting algorithm.

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Chapter 7

Models of liner shipping network design

This chapter is a discussion of models for the LSNDP investigating possibilities for strengthening current formulations of the problem. The discussion is inspired by state-of-the-art within Multicommodity Capacitated Network Design problems (hereafter denoted *MCND*) and Pick-up-and-Delivery Vehicle Routing Problems (PDVRP) with split deliveries, which contains similarities to the LSNDP problem. The discussion is motivated by the fact that the LSNDP may be modeled as a specialized variant of MCND often displaying a higher degree of freedom for the flow. As a result some of the known cutting planes and methods developed for MCND cannot be exploited in current models of the LSNDP. The LSNDP can also be seen as a rich formulation of the PDVRP with split deliveries without designated depots. The latter complicates the use of existing methods developed for the PDVRP with time windows and Split Deliveries (Stålhane et al., 2012).

In Chapter 2 a reference model for the LSNDP was proposed. Providing a reference model for the benchmark suite is important, because nearly every journal paper on the LSNDP provides a different model of the LSNDP. In order to compare algorithms on a given benchmark set it is important to agree on an objective function and a set of constraints, which reflect real life operation of global liner shipping companies. However, the formulation is not a compact formulation suitable for exact methods. The model is based on the set of possible rotations combining different port calls with a vessel class and an average speed. This yields an exponential number of variables. The most common technique to overcome this problem is to apply delayed column generation in a ranch-and-Price context, where the reference model would be the Master problem, and a pricing problem would be applied to generate the route variables based on their reduced cost in the master problem. The drawback of the reference model for applying this technique is that each column is associated with specific rows defining the capacity of the arcs sailed. This complicates the use of a delayed column generation technique, as columns are evaluated according to their reduced cost and the optimality condition is based on proving that all non-basic variables have a non-negative reduced cost in the case of a minimization problem. The addition of a variable requires addition of a new set of rows meaning that the calculation of the reduced cost of a column must be reevaluated after adding it to the master problem. It also affects the optimality criterion as rows exist, which are not present in the master problem at the time of evaluation. Therefore, it is still important to explore alternative ways of formulating the LSNDP as a mathematical program in order to achieve a strong and compact formulation for the purpose of developing exact methods exploiting state-of-the art techniques within mathematical programming.

This Chapter explores how to strengthen mathematical models of the LSNDP and identifies the major challenges in constructing a strong mathematical formulation for the LSNDP. The extensive research done on the polyhedra of MCNDs and the decomposition methods, that constitute state-of-the-art exact solution methods for both MCND and VRPs are used as a reference and inspiration to the discussion in this chapter. A model inspired by PDVRP with Split Deliveries is presented

at the end of the Chapter. The model is not a perfect, standardized mathematical model for the LSNBP and it is not trivial to design a Branch-and-Cut-and-Price algorithms for the model given a complex pricing problem. Heuristics can be designed and research into relaxations of the pricing problem is needed to obtain good bounds for the LP relaxation. In this way the model can contribute to design a mathematical formulation for the LSNBP, which is suitable for a Branch-and-Cut-and-Price algorithm to solve smaller instances of the LSNBP.

7.1 Scope of LSNBP models in this thesis

The classification scheme of Kjeldsen (2011) gives a good overview of the key components in routing and scheduling problems of liner shipping. However, the definition of routing and scheduling problems in liner shipping by Kjeldsen (2011) is broader than the definition of LSNBP as it is understood in this thesis. The scope of the thesis, and hence also the models discussed in this thesis, can be related to the classification scheme using the characteristics presented in Table 7.1.

No.	Characteristic	Options
1	Number of starting Points	None
2	Type of operation	Pick-up and delivery interwoven
3	Nature of Demand	Deterministic
6	Fleet composition	Heterogeneous
8	Demand Splitting	Allowed
10	Number of Capacity Types	Multiple
11	Cargo Transshipment	Allowed
14	Ships required to be empty	No

Table 7.1: Classification of the models considered in this thesis in the terms used in Kjeldsen (2011). Only terms, where a certain option is required are described.

In this thesis models of liner shipping network design refers to the determination of multiple routes for container vessels to provide network wide transportation of multiple commodities. Within the scope defined for the LSNBP there are several models differing on the cost of transshipping, if there is a scheduling component (Agarwal and Ergun, 2008), weekly frequency (Agarwal and Ergun, 2008; Jepsen et al., 2011) or not (Alvarez, 2009; Reinhardt and Pisinger, 2011). The challenges occurring when transferring known methods from the MCND and the VRP problems are mainly in the area of *characteristic 11* described in table 7.1, where cargo transshipments are allowed and *characteristic 14*, where ships are not required to be empty at any point along the route. Note that, in the case where transshipment costs must be charged, it is even more complicated.

7.2 Relation to capacitated network design problems

Models adhering to the classification in Table 7.1 may be seen as a special case of the classical MCND defined for a directed graph $G = (N, A)$ and a set of commodities K . Each commodity is defined by its origin O_k and destination D_k and a volume to be transported d_k . For each arc (i, j) there is a fixed cost f_{ij} of assigning capacity to the arc and for each commodity and arc, the cost, c_{ij}^k , denotes the transportation cost for a unit of commodity k on arc (i, j) . Each arc has a capacity of u_{ij} and the in-and-out going arc sets of a node i is defined as $N^+(i) = \{(j \in N : (i, j) \in A)\}$, $N^-(i) = \{(j \in N : (j, i) \in A)\}$. Continuous variables, x_{ij}^k , are defined to model the cargo flow and integer design variables, y_{ij} , are defined for each arc to model, where capacity is assigned. An arc-flow formulation of the MCND is stated below:

$$\text{Min} \quad \sum_{k \in K} \sum_{(ij) \in A} c_{ij}^k x_{ij}^k + \sum_{(ij) \in A} f_{ij} y_{ij} \quad (7.1)$$

$$\text{s.t.} \quad \sum_{j \in N^+(i)} x_{ij}^k - \sum_{j \in N^-(i)} x_{ji}^k = \begin{cases} -d_k & \text{if } i = O_k, \\ 0 & \text{if } i \in A \setminus \{O_k, D_K\} \\ d_k & \text{if } i = D_k. \end{cases} \quad \forall i \in N \quad \forall k \in K \quad (7.2)$$

$$\sum_{k \in K} x_{ij}^k \leq u_{ij} y_{ij} \quad \forall (i, j) \in A \quad (7.3)$$

$$x_{ij}^k \geq 0 \quad \forall (i, j) \in A \quad \forall k \in K \quad (7.4)$$

$$y_{ij} \in \{0, 1\} \quad \forall (i, j) \in A \quad (7.5)$$

The objective function (7.1) minimizes the transportation cost of the flow and the asset cost of providing capacity on arcs. Constraints (7.2) are the *mass balance constraints* (Ahuja et al., 1993) for each commodity and constraints (7.3) are the capacity constraints for the capacity provided matched with the commodity flow for each arc. If the y_{ij} variables are fixed, a linear multicommodity flow problem defined for the resulting graph remains. The relation to LSNDP concerns the general setup with a two tier structure of asset cost at the upper tier and operational cost of flow at the second tier. There is a vast body of literature on the MCND, e.g., the comprehensive survey Gendron et al. (1999), the article of Stoer and Dahl (1994) and the more recent paper of Frangioni and Gendron (2009). The MCND is considered a challenging optimization problem from the perspective of model and algorithmic development (Gendron et al., 1999). The problem requires advanced solution techniques to solve even moderate sized instances of the problem to optimality. There are different aspects as to why the MCND is considered hard to solve. First and foremost, the integrality gap is often prohibitively large. The linear relaxation of the MCND becomes highly fractional, as the linear multicommodity flow problem will prefer to provide exactly the fraction of all capacities needed to meet the demand requirements (Gendron et al., 1999). Therefore, the lower bounds for the Branch-and-Bound methods are often poor resulting in very large Branch-and-Bound trees. As a result extensive research has been done to strengthen the bounds using valid inequalities and cutting planes (Atamtürk, 2002; Costa et al., 2009; Günlük, 1999, e.g.). The problem of high fractionality is severe for models of the LSNDP due to economies of scale on container vessels guiding fractions of the largest container vessel onto all arcs (Alvarez, 2009; Plum, 2010). In models of the LSNDP with the objective of maximising revenue with partial demand satisfaction (Agarwal and Ergun, 2008; Alvarez, 2009; Plum, 2010) the problem is even more explicit because a revenue is obtained in the lower tier, resulting in differing objectives for the flow and the capacity installation. Secondly, the linear multicommodity flow problem scales poorly due to the vast amount of arc flow variables. The LSNDP is a special case of the MCND as the following constraints are added to the classical formulation:

1. The capacity assigned constitute vessel routes in the form of cycles also known as design balance requirements, $\sum_{j \in N^+(i)} y_{ij} = \sum_{j \in N^-(i)} y_{ji}$.
2. A heterogeneous fleet of vessels results in a discrete set of capacities available.
3. The model and/or graph topology will be able to detect and price transshipments of cargo.

Specialised MCND models with the first and second requirement are common within airline, e.g. Hane et al. (1995), and telecommunication network design, e.g., Dahl and Stoer (1998), Uster and Kumar (2010), but the third requirement is not as common. In Reinhardt and Pisinger (2011) the transshipment cost, which is a significant part of the total network cost, is modelled by using a distinct network layer for each vessel in the fleet. This modelling technique does not lend itself well to LSNDP models applying a weekly frequency and hence deploying several vessels to a single route.

7.3 Strengthening formulations of LSNDP

The complexity of the LSNDP is partly due to high degrees of freedom resulting in an enlarged solution space compared to the classical MCND. The network consists of non-simple cyclic routes, partial demand satisfaction is allowed, and all commodities are allowed to transship (Characteristic 8 and 11 in Table 7.1). The following Section is a discussion of techniques, constraints and requirements, which may be used to strengthen formulations of the LSNDP.

7.3.1 Cutting planes for capacitated network design related to LSNDP

Cutting planes, in particular the cut-set inequalities, for the MCND are well studied, e.g., Atamtürk (2002); Costa et al. (2009); Günlük (1999) and Stoer and Dahl (1994). Computational results from Atamtürk (2002); Costa et al. (2009) and Günlük (1999) show that adding cut-set inequalities result in better bounds at the root node and fewer nodes in the search tree, due to the tighter bounds. A single cutting plane algorithm has been presented for the LSNDP (Reinhardt and Pisinger, 2011), however, not using the cut-set inequalities. In the case of Reinhardt and Pisinger (2011) cut-set inequalities may be used to strengthen the formulation as partial satisfaction of demand is not allowed. In many models of the LSNDP partial satisfaction of demand is allowed and hence metric, cut-set and Benders inequalities cannot be applied. Future models of the LSNDP allowing partial demand satisfaction could be strengthened by distinguishing between optional and “contracted” cargo as seen in the case of tramp-shipping (Christiansen et al., 2007). The distinction is desirable from a carriers point of view as cargo contracts do exist between carriers and key clients as described in Section 2.4.4 on page 31. These contracts are not legally binding for neither carrier nor shipper, but are a sign of a mutual commitment. Therefore, it is good practise to ensure that the capacity needed to fulfill the contracts is available. The carrier may also have a strategic position on a particular trade, which requires carrying a significant volume on a trade. Partial demand satisfaction may also be in conflict with the carriers value position. However, not allowing partial demand splitting, may force vessels into regions with limited demand, and of minor strategic relevance to the carrier, at a very high cost. This means that some flexibility is required, but it would be beneficial to establish a minimum bound on the demand transported for most commodities. This minimum bound may be used to introduce cut-set inequalities for the lower bound on demand satisfaction and thus contribute to strengthening models of the LSNDP.

7.3.2 Variable aggregation

In order to reduce the number of arc flow variables for the linear multicommodity flow problem a common method is to aggregate variables by either origin or destination as seen in Agarwal and Ergun (2008); Alvarez (2009). However, the aggregation and reduction of arc flow variables may come at a high price in terms of closing the integrality gap. In Croxton et al. (2007) computational experiments on formulations of the MCND with and without aggregation of variables show that the aggregation may result in worse LP-bounds and therefore, there is a trade off between a leaner formulation of the flow through the network and the integrality gap, which must be closed by the Branch-and-Bound method used to obtain an integer solution.

The aggregation of variables presented in Chapter 6 is another way of reducing the number of variables was introduced, in order to reduce problem size without deteriorating the integrality gap, but computational results must be performed to evaluate the aggregation method in relation to the integrality gap.

7.3.3 Level of service requirements

From a business perspective introducing level of service is crucial to construct a competitive liner shipping network. Level of service requirement enforces a maximum transit time for each commodity and hence limits the number of allowed paths in the network for the individual commodity.

It requires adding a time dimension to each arc in the network with special consideration for the time spent transshipping between services.

Introducing maximum transit time limits is not trivial in arc flow models of a MCND variant, as it is hard to keep track of the individual paths of commodities if you allow a demand to travel via several paths from origin to destination (in the Origin-Destination Integer Multicommodity Flow Problem (Barnhart et al., 2000), level of service would be easier to model as it allows for a single path per commodity, but it is \mathcal{NP} -hard to solve just for the flow). In a path-flow formulation a time-dimension may be added to the paths, but it requires the linear multicommodity flow problem to be decomposed into a path-flow formulation solved by delayed column generation as seen in Brouer et al. (2011). The pricing problem generating paths for commodities has added complexity, because it requires a resource-constrained shortest path problem to find a time feasible path with the best reduced cost. The resource constrained shortest path problem is weakly \mathcal{NP} -hard and can be solved in pseudo-polynomial time with a label setting algorithm (Irnich and Desaulniers, 2005).

It is not trivial to see whether requiring level of service will tighten the formulation and decrease the integrality gap as the time requirements constrain the feasible flow, but are only implicitly constraining the integer variables. Level of service requirements are important to pursue in future models of the LSNDP to evaluate, whether they improve the integrality gap or merely complicate solving the linear relaxation. In either case they describe an additional complexity of a real life liner shipping network design problem.

7.3.4 Empty container repositioning through load balancing

Empty repositioning is an important part of the traffic flow in any liner shipping network and empty loads constitute a significant part of all container transports as discussed in Chapter 3. It is possible to account for the empty flows by adding interbalancing constraints to the multicommodity flow problem solved in the LSNDP models. An empty commodity must be introduced with a variable for each arc to account for the empty flows in models similar to MCND. Interbalancing constraints are introduced for each node of the network requiring flow into a node to equal flow out of the node constituted by commodities and the empty flow. The computational experiments presented in Chapter 3 indicate, that this can readily be incorporated, when solving the flow. The requirement, to balance flow at nodes, constrains the solution space of the multicommodity flow problem. The number of non-integer variables will increase by the number of edges, while the constraint set will increase by the size of the nodes. Like level of service requirements empty repositioning constrains the linear multicommodity flow problem, and implicitly constrains capacity. It is uncertain, whether it will improve upon the integrality gap.

7.4 Path-based decomposition of the LSNDP

Chapter 6 presented a model of the LSNDP constructing routes with cargo loading patterns inspired by the models developed within various Vehicle Routing Problems (VRP). Modelling the LSNDP inspired by VRP is complicated by both the non-simple cyclic routes, with no predefined starting point, where vessels are not required to be empty at any point along a route (Characteristic 14 in Table 7.1) as well as having multiple commodities allowing for transshipments (Characteristic 8 and 11 in Table 7.1).

The idea of the model is to construct service variables that carry both cost and revenue to decrease the integrality gap. Moreover a new approach to aggregate the number of explicit demands is formulated rooted in the perception of trades between regions instead of specific origin-destination pairs in liner shipping. In this way it reduces the number of commodities in a novel way, which is close to the perception of trades in liner shipping. The model is suitable for Dantzig-Wolfe Decomposition. The project explores the possibilities of constructing a Branch-and-Price algorithm on a compact formulation with demand aggregation to get a coarse network design. The network design can subsequently be refined by new models and methods not unlike the sequential approach to

network design seen in the airline industry. The model resembles the PDVRPTW with split loads described in Stålhane et al. (2012). In the paper profit is maximised for transporting contracted and optional cargoes using a fleet of heterogeneous vessels. The paper expands the label-setting algorithms and dominance criteria for the Pick-up-And-Delivery Vehicle Routing Problem with Time Windows (PDVRPTW) (Ropke and Cordeau, 2009) and for the Split-Delivery Vehicle Routing Problem with Time Windows (SDVRPTW) (Desaulniers, 2010). The sub-problem returns a route and delivery pattern that violates the optimality condition the most by solving a resource constrained shortest path problem for the route combined with a multidimensional linear knapsack problem to determine the cargo quantities. In the model of Jepsen et al. (2011) transshipments are allowed and unlike Stålhane et al. (2012) routes are cyclic and cannot be assigned a starting point or an artificial ending point, meaning that the vessel is not guaranteed to be empty at any point. These properties add to the complexity of the pricing problem as the usage of delivery pattern variables cannot be directly transferred to this model, because the calculation of the reduced cost in Desaulniers (2010) and Stålhane et al. (2012) is calculated once the truck/vessel is empty. As a consequence solving the pricing problem using dynamic programming and a label setting algorithm does not make use of delivery patterns and apply discretization of all cargo quantities to ensure optimality. This gives rise to a combinatorial explosion in the pricing problem, due to the large scale of each demand in units. Furthermore, as there is no starting point, there are no precedence relations on the pick-ups and deliveries of separate commodities, which is normally exploited in variants of the PDVRP. Dominance criteria are weak for the discretization resulting in a near brute-force method generating all paths and cargo combinations in the label setting algorithm. The implementation of the label setting algorithm showed disappointing results. Various relaxations of the pricing problem or alternative solution methods maybe pursued in order to obtain upper bounds for a Branch-and-price algorithm, or use the model to pursue a heuristic column generation procedure.

The model in Jepsen et al. (2011) is based on a decomposition of the set of services. This set contains an exponential number of combinations of a set of port calls, which may constitute a non-simple cycle. The next section presents a model inspired by Jepsen et al. (2011), but with a vessel class based decomposition. The set of vessel classes is finite and significantly smaller than the set of services. The number of services possible given a finite number of vessels available in the vessel class should be more tractable. The model decomposed by vessel class yields a new sub-problem of multiple cycles with associated extreme cargo patterns modelled in the same way as Desaulniers (2010); Stålhane et al. (2012). Solving the pricing problem to optimality remains an open research question. Heuristic methods, which can be used to obtain bounds, are discussed at the end of this Chapter. However, the algorithm has not been implemented and hence, we do not yet know, whether these approaches are sufficient to make the problem computationally tractable for small instances.

7.4.1 Path based model decomposing by vessel class

The path based model in Jepsen et al. (2011) is a Dantzig-Wolfe Decomposition of the arc-flow model in Løfstedt et al. (2011). The model presented in this section is a reformulation of this model, where the set of services is replaced by the set of the vessel classes. The set of services is defined within each vessel class. In this model demand is not aggregated. It is possible to transform the model to use the aggregation from Jepsen et al. (2011). The model presented here is joint work with Guy Desaulniers, GERAD, Montréal.

The model presented here is still work in progress and is motivated by the difficulties of solving the pricing problem discussed in Chapter 6. The pricing problem returns a set of routes, which together deploys the vessels available in the individual vessel class. The model has a drawback as the exponential set of services are represented using a variable for each subset of nodes, which is decomposed and occurs in the pricing problem subsequently. However, we relax the pricing problem to solve a set of routes without these subset variables. As a consequence the cycles returned from the pricing problem may each deploy a fractional number of vessels and the reduced cost must be reevaluated for the rounded up integer number of vessels.

7.4.2 Notations

Here we define the sets and parameters used in this model:

- A is the finite set of vessel classes considered with capacity $C^a, a \in A$.
- P is the set of ports.
- V is the set of port vertices in the graph. Several vertices may exist for a single port $p \in P$ to model non-simple routes. Each vertex i is associated with a port p by function $p(i)$.
- $H_p \subset V$ is the set of port vertices belonging to port p .
- K is the set of commodities. A commodity is defined by its origin port $O_k \in P$, its destination $D_k \in P$ and a demand $b^k, k \in K$.
- G_k is the set of ports allowing transshipment of commodity $k \in K$.
- MIN^k is a minimum required delivery of a commodity $k \in K$.
- z^k is the revenue of one unit of commodity $k \in K$.
- l_p is the cost to load one unit of flow in port p .
- u_p is the cost to unload one unit of flow in port p .
- c_p^a is the cost to visit port p with vessel class a
- c_v^a is the weekly cost for a vessel of class a .
- $E_p^a \in \{0, 1\}$ is set iff vesselclass a is not draft restricted in port p .
- W^a is the weekly distance covered by a vessel of class a at a predefined slow steaming speed.
- N^a is the number of vessels of vessel class a in the fleet.
- d_{qp} is the distance in nautical miles from port q to p .
- $\delta(S) = \sum_{i \in S} \sum_{j \in S} x_{ij}$ is the set of internal edges in $S \subset V$.

The **variables** used in the model are as follows:

- L_{ia}^k is the amount of containers of commodity $k \in K$ loaded in port vertex $i \in V$ by vessel class a . The variables are defined for $i \in O_k \cup G_k$
- U_{ia}^k is the amount of containers of commodity $k \in K$ unloaded in port vertex $i \in V$ by vessel class a . The variables are defined for $i \in D_k \cup G_k$
- $x_{ij}^a \in \{0, 1\}$ is a decision variable indicating whether vessel class a will sail between port vertex i and port vertex j .
- f_{ia}^k is the amount of containers of commodity k carried on vessel class a in port vertex i .
- $\lambda_S^a \in \{0, 1\}$ is a decision variable indicating if $S \subseteq V$ is the minimal set containing a single cycle of all subsets $S' \subseteq S$ of S for vessel class a .
- $n_S^a \in \mathbb{Z}^+$ is an integer variable counting the minimal number of vessels of class a needed for a weekly frequency of the cycle defined in $S \subseteq V$.

An arc-flow mathematical model is presented below:

$$\max \sum_{a \in A} \sum_{k \in K} \sum_{i \in O_k} z^k \cdot L_{ia}^k \quad (7.6)$$

$$- \sum_{a \in A} \sum_{k \in K} \sum_{i \in V} (l_{p(i)} L_{ia}^k + u_{p(i)} U_{ia}^k) \quad (7.7)$$

$$- \sum_{a \in A} \sum_{i \in V} \sum_{j \in V} c_{p(i)}^a x_{ij}^a \quad (7.8)$$

$$- \sum_{a \in A} \sum_{S \subseteq V} n_S^a c_v^a \quad (7.9)$$

The objective function may be split into 4 terms. (7.6) is the revenue of container transport, (7.7) are the cargo handling expenses including transshipment cost, (7.8) are the port call costs and finally (7.9) is the cost of vessels deployed.

The constraints for the flow of multiple commodities are stated below:

$$MIN^k \leq \sum_{a \in A} \sum_{i \in O_k} L_{ia}^k \leq b^k \quad \forall k \in K \quad (7.10)$$

$$\sum_{a \in A} \sum_{i \in O_k} L_{ia}^k = \sum_{a \in A} \sum_{i \in D_k} U_{ia}^k \quad \forall k \in K \quad (7.11)$$

$$\sum_{a \in A} \sum_{i \in H_p} (L_{ia}^k - U_{ia}^k) = 0 \quad \forall p \in G_k, \forall k \in K \quad (7.12)$$

$$\sum_{i \in V} L_{ia}^k = \sum_{i \in V} U_{ia}^k \quad \forall k \in K, \forall a \in A \quad (7.13)$$

Constraints (7.10), (7.11) and (7.12) ensure load balance for each commodity for loads, unloads and transshipments. Constraints (7.13) ensure that everything loaded on a vessel class is unloaded from the vessel class. The constraints are redundant with (7.10), (7.11) and (7.12), but are needed in the pricing problem when decomposing. A minimum delivery of each commodity is imposed in order to enable the use of cut-set inequalities for the “contracted” demands. The flow is further constrained by a designated set, G_k , of transshipment ports for each cargo $k \in K$. This limits the number of possible paths for each commodity and forces transshipments onto the main ports identified for an instance. The constraint is realistic as port have a certain productivity in terms of the cranes available for a given vessel and also the efficiency of the cranes. Transshipping cargo at feeder ports (spokes) is not desirable simply due to smaller ports being less effective at performing transshipments.

The constraints defining the cycles and the number of vessels deployed to each cycle is defined as:

$$\sum_{(i,j) \in \delta(S)} x_{ij}^a \leq |S| - 1 + \sum_{S' \subseteq S} \lambda_{S'}^a \quad \forall S \subseteq V, 2 \leq |S| \leq |V|, \forall a \in A \quad (7.14)$$

$$n_S^a \geq \sum_{(i,j) \in \delta(S)} x_{ij}^a \frac{d_{p(i)p(j)}}{W^a} - |N^a|(1 - \lambda_S^a) \quad \forall S \subseteq V, 2 \leq |S| \leq |V|, \forall a \in A \quad (7.15)$$

$$\sum_{S \subseteq V} n_S^a \leq N_a \quad \forall a \in A \quad (7.16)$$

$$\sum_{j \in V} x_{ij}^a - \sum_{j \in V} x_{ji}^a = 0 \quad \forall i \in V, \forall a \in A \quad (7.17)$$

$$\sum_{j \in V} x_{ij}^a \leq 1 \quad \forall i \in V, \forall a \in A \quad (7.18)$$

Constraints (7.14) and (7.15) are used to detect the integer number of vessels needed for each cycle. (7.14) sets λ_S^a only for the minimal set containing a cycle in all port vertex subsets of S

and (7.15) sets n_S^a to the minimum integer number of vessels needed to complete the cycle with the distance at a weekly frequency. The weekly frequency constraint limits the number of vessels applied in a vessel class in (7.16). Cyclic rotations are ensured by (7.17) and constraints (7.18) ensure that each port vertex $i \in V$ can only be visited once. The definition of the port vertex set V contains $|H_p|$ duplicates of port $p \in P$ and hence multiple port visits and non-simple cyclic services are allowed.

In order to correctly calculate the cargo patterns respecting the capacity of the vessel for each arc on its voyage we define:

$$L_{ia}^k - C^a \sum_{j \in V} x_{ij}^a \leq 0 \quad \forall k \in K, \forall i \in O_k \cup G_k, \forall a \in A \quad (7.19)$$

$$U_{ia}^k - C^a \sum_{j \in V} x_{ij}^a \leq 0 \quad \forall k \in K, \forall i \in D_k \cup G_k, \forall a \in A \quad (7.20)$$

$$\sum_{k \in K} f_{ia}^k \leq C^a \sum_{j \in V} x_{ij}^a \quad \forall i \in V, \forall a \in A \quad (7.21)$$

$$f_{ja}^k \geq f_{ia}^k + L_{ja}^k - U_{ja}^k - 2C^a(1 - x_{ij}^a) \quad \forall i \in V, \forall a \in A \quad (7.22)$$

Constraints (7.19)-(7.20) limit load and unload to visited port vertices. Constraints (7.21) ensure that a vessel class does not exceed its capacity at departure from a port. Constraints (7.22) sets the flow variables for the entire cycle.

$$0 \leq L_{ia}^k \leq b^k \quad \forall i \in V, p(i) = O_k \cup G_k \quad \forall k \in K \quad \forall a \in A \quad (7.23)$$

$$0 \leq U_{ia}^k \leq b^k \quad \forall i \in V, p(i) = D_k \cup G_k \quad \forall k \in K \quad \forall a \in A \quad (7.24)$$

$$x_{ij}^a \in \{0, 1\} \quad \forall i \in V, \forall j \in V, \quad \forall a \in A, E_{p(i)}^a = 0 \wedge E_{p(j)}^a = 0 \quad (7.25)$$

$$f_{ia}^k \in \mathcal{R}^+ \quad \forall i \in V \forall k \in K \quad \forall a \in A \quad (7.26)$$

$$\lambda_S^a \in \{0, 1\} \quad \forall S \subseteq V, \quad \forall a \in A \quad (7.27)$$

$$n_S^a \in \mathcal{Z}^+ \quad \forall S \subseteq V, \quad \forall a \in A \quad (7.28)$$

Constraints (7.23)-(7.28) define the domains of the variables.

7.4.3 Dantzig-Wolfe decomposition

Model (7.6)-(7.28) has block angular structure and may be decomposed according to the Dantzig-Wolfe decomposition principle. Each separable block define a set of constraints as the sub-problem domain. Extreme points of this domain can be used to reformulate the original linear program into a master problem with a reduced set of constraints as only the linking constraints between blocks remain in the master problem. In the master problem the variables are constituted by all the extreme points and rays of the sub-problem domain. The block-angular structure is given by constraints (7.13)-(7.28) separable by vessel class resulting in one sub-problem for each vessel class. Define the feasible domain of the sub-problem for vessel class a as

$$D^a = \{(x^a, L^a, U^a, \lambda^a, n^a) | (x^a, L^a, U^a, \lambda^a, n^a) \text{ satisfies constraints (7.13) - (7.28) for vessel class } a \in A\} \quad (7.29)$$

Let the sets

$$S = \{(i, j) \in \delta(S), L_{ia}^k, U_{ia}^k, x_{ij}^a, n_S^a | \lambda_S^a = 1\} \quad (7.30)$$

define a cycle r with a loading pattern, a number of vessels deployed, and a speed. The speed is predefined as 30-40% above slow steaming speed as in 6. An extreme point of domain D^a corresponds to a set of cycles R^a , where the total number of vessels deployed in R^a cannot exceed $|N^a|$. There are no extreme rays of domain D^a as (7.13)-(7.28) is bounded.

The set of cycles defining an extreme point of D^a has a special property due to the fact that constraint (7.16) is not tight: Each cycle r in the set of cycles R^a constitute an extreme point since it satisfies constraints (7.13)-(7.28) for vessel class c . When reformulating the master problem we will exploit this property as the variables returned by the sub-problem will constitute a single cycle $r \in R^a$. In order to reduce the number of variables in the problem and reduce symmetry in the problem a route is combined with a set of extreme feasible loading patterns W_r to avoid discretization of cargo flows. This is due to the fact that once (x_r^a, n_r^a) are fixed for a given set of port vertices, where $\lambda_S^a = 1$ there are several feasible (and extreme) loading patterns consisting of (L_r^a, U_r^a) respecting the capacity constraint of the vessel. Determining the set W_r is a separate optimization problem. In the solution of the master problem a single cycle may be associated with a linear combination of the set of extreme feasible loading patterns $w \in W_r$.

The following definitions are introduced for the master problem:

- R^a is the set of cycles of vessel class a .
- $y_r^a \in \{0, 1\}$ is a binary variable indicating whether cycle r for vessel class a is selected in the solution of the master problem.
- $y_{rw}^a \in \mathcal{R}$ is a non-negative continuous variable indicating the fraction of loading pattern w associated to cycle r for vessel class a in the solution of the master problem.
- α_{irw}^{ka} is the amount of containers of commodity $k \in K$ loaded in loading pattern w for cycle r at port vertex $i \in V$ on vessel class a .
- β_{irw}^{ka} is the amount of containers of commodity $k \in K$ unloaded in loading pattern w for cycle r at port vertex $i \in V$ on vessel class a .
- $n_r^a \in \mathbb{Z}^+$ is an integer denoting the number of vessels of class a deployed on cycle r .
- $a_{ir} \in \{0, 1\}$ is a binary parameter equal to one iff port vertex $i \in V$ is visited by cycle r .

Let the cost of a cycle $r \in R^a$ be defined as:

$$c_{rw}^a = \sum_{k \in K} \sum_{i \in O_k} z^k \alpha_{irw}^{ka} - \sum_{i \in V} \sum_{k \in K} (l_{p(i)} \alpha_{irw}^{ka} + u_{p(i)} \beta_{irw}^{ka}) - \sum_{i \in V} a_{ir} c_{p(i)}^a - n_r^a c_v^a$$

This leads us to the decomposed model, where each column represents a rotation with a cargo (un)loading pattern and an integer number of vessels deployed :

$$\text{Max} \quad \sum_{a \in A} \sum_{r \in R^a} \sum_{w \in W_r} c_{rw}^a y_{rw}^a \quad (7.31)$$

$$\text{s.t} \quad \text{MIN}^k \leq \sum_{a \in A} \sum_{r \in R^a} \sum_{w \in W_r} \sum_{i \in O_k} \alpha_{irw}^{ka} y_{rw}^a \leq b^k \quad \forall k \in K \quad (7.32)$$

$$\sum_{a \in A} \sum_{r \in R^a} \sum_{w \in W_r} \left(\sum_{i \in O_k} \alpha_{irw}^{ka} - \sum_{i \in D_k} \beta_{irw}^{ka} \right) y_{rw}^a = 0 \quad \forall k \in K \quad (7.33)$$

$$\sum_{a \in A} \sum_{r \in R^a} \sum_{w \in W_r} \sum_{i \in H_p} (\alpha_{irw}^{ka} - \beta_{irw}^{ka}) y_{rw}^a = 0 \quad \forall p \in G_k, \forall k \in K \quad (7.34)$$

$$\sum_{r \in R^a} n_r^a y_r^a \leq N_c \quad \forall a \in A \quad (7.35)$$

$$y_r^a = \sum_{w \in W_r} y_{rw}^a \quad \forall r \in R^a, \forall a \in A \quad (7.36)$$

$$y_{rw}^a \geq 0 \quad \forall r \in R^a, \forall w \in W_r \quad (7.37)$$

$$y_r^a \in \{0, 1\} \quad \forall r \in R^a, \forall a \in A \quad (7.38)$$

Objective (7.31) corresponds to (7.6)-(7.9). Constraints (7.32)-(7.34) corresponds to (7.10)-(7.12). Constraints (7.37) enforce non-negativity on the loading patterns and the y_r^a variables are defined in terms of the y_{rw}^a variables in constraints (7.36). Constraint (7.35) is the generalized convexity constraint of domain D^a . The convexity constraint has been relaxed due to the special property of the feasible domain, where each individual cycle constitutes an extreme point in itself. Normally the convexity constraints are required to sum to one, but in this case the convexity constraint limits the deployment of vessels in the set of cycles to the vessels available in vessel class a .

7.4.4 Pricing problem

To solve the master problem using delayed column generation it is desirable to add columns consisting of a route with a feasible extreme cargo pattern for each vessel class with positive reduced cost. The pricing problem for a vessel class a consist of constraints (7.13)-(7.28) with the objective of maximizing the reduced cost for the vessel class a . To calculate the reduced cost we define the following dual variables:

- Let ω^k be the duals of constraints (7.32) defined for each commodity $k \in K$. Please note that there are two separate constraints per commodity k , but either one might be binding and hence it will be positive or negative depending on which is binding. $\omega^k \in \mathcal{R}$ is hence a free variable.
- Let θ^k be the duals of constraints (7.33) defined for each commodity $k \in K$. $\theta^k \in \mathcal{R}$ is a free variable.
- Let $\gamma_p^k \in \mathcal{R}$ be the dual variable of constraint (7.34) defined for each commodity $k \in K$ and for each transshipment port $p \in G_k$. γ_p^k is likewise a free variable.
- Let $\pi^a \in \mathcal{R}+$ be the dual variable of constraint (7.35) for vessel class $a \in A$.

The reduced cost of the variable y_{rw}^a is

$$\hat{c}_{rw}^a = c_{rw}^a - \sum_{k \in K} \sum_{i \in O_k} \alpha_{irw}^{ka} \omega^k - \sum_{k \in K} \left(\sum_{i \in O_k} \alpha_{irw}^{ka} - \sum_{i \in D_k} \beta_{irw}^{ka} \right) \theta^k - \sum_{k \in K} \sum_{p \in G_k} \sum_{i \in H_p} (\alpha_{irw}^{ka} - \beta_{irw}^{ka}) \gamma_p^k - n_r^a \pi^a$$

Expanding the term c_{rw} we get the following reduced cost for y_{rw}^a :

$$\begin{aligned} \hat{c}_{rw}^a &= \sum_{k \in K} \sum_{i \in O_k} z^k \alpha_{irw}^{ka} - \sum_{i \in V} \sum_{k \in K} (l_{p(i)} \alpha_{irw}^{ka} + u_{p(i)} \beta_{irw}^{ka}) - \sum_{i \in V} a_{ir} c_{p(i)}^a - n_r^a c_v^a \\ &\quad - \sum_{k \in K} \sum_{i \in O_k} \alpha_{irw}^{ka} \omega^k - \sum_{k \in K} \left(\sum_{i \in O_k} \alpha_{irw}^{ka} - \sum_{i \in D_k} \beta_{irw}^{ka} \right) \theta^k \\ &\quad - \sum_{k \in K} \sum_{p \in G_k} \sum_{i \in H_p} (\alpha_{irw}^{ka} - \beta_{irw}^{ka}) \gamma_p^k - n_r^a \pi^a \\ &= \sum_{k \in K} \sum_{i \in O_k} (z^k - l_{p(i)} - \omega^k - \theta^k) \alpha_{irw}^{ka} - \sum_{k \in K} \sum_{i \in D_k} (u_{p(i)} + \theta^k) \beta_{irw}^{ka} \\ &\quad - \sum_{k \in K} \sum_{p \in G_k} \sum_{i \in H_p} ((l_{p(i)} + \gamma_p^k) \alpha_{irw}^{ka} - (u_{p(i)} - \gamma_p^k) \beta_{irw}^{ka}) \\ &\quad - \sum_{i \in V} a_{ir} c_{p(i)}^a - (c_v^a + \pi^a) n_r^a \end{aligned}$$

To find variables y_{rw}^a with a positive reduced cost we define a pricing problem for vessel class $a \in A$ as follows:

$$\begin{aligned}
Max \sum_{k \in K} \sum_{i \in O_k} (z^k - l_{p(i)} - \omega^k - \theta^k) L_i^k - \sum_{k \in K} \sum_{i \in D_k} (u_{p(i)} + \theta^k) U_i^k \\
- \sum_{k \in K} \sum_{p \in G_k} \sum_{i \in H_p} ((l_{p(i)} + \gamma_p^k) L_i^k - (u_{p(i)} - \gamma_p^k) U_i^k) \\
- \sum_{i \in V} \sum_{j \in V} c_{p(i)}^a x_{ij} - \sum_{S \subseteq V} (c_v^a + \pi^a) n_S
\end{aligned} \tag{7.39}$$

$$s.t. \sum_{i \in V} L_i^k = \sum_{i \in V} U_i^k \quad \forall k \in K \tag{7.40}$$

$$\sum_{(i,j) \in \delta(S)} x_{ij} \leq |S| - 1 + \sum_{S' \subseteq S} \gamma_{S'} \quad \forall S \subseteq V, 2 \leq |S| \leq |V| \tag{7.41}$$

$$n_S \geq \sum_{(i,j) \in \delta(S)} x_{ij} \frac{d_{p(i)p(j)}}{W^a} - N^a (1 - \gamma_S) \quad \forall S \subseteq V, 2 \leq |S| \leq |V| \tag{7.42}$$

$$\sum_{S \subseteq V} n_S \leq N_c \tag{7.43}$$

$$\sum_{j \in V} x_{ij} - \sum_{j \in V} x_{ji} = 0 \quad \forall i \in V \tag{7.44}$$

$$\sum_{j \in V} x_{ij} \leq 1 \quad \forall i \in V \tag{7.45}$$

$$L_i^k - C^a \sum_{j \in V} x_{ij} \leq 0 \quad \forall k \in K, \forall i \in O_k \cup G_k \tag{7.46}$$

$$U_i^k - C^a \sum_{j \in V} x_{ij} \leq 0 \quad \forall k \in K, \forall i \in D_k \cup G_k \tag{7.47}$$

$$\sum_{k \in K} f_i^k \leq C^a \sum_{j \in V} x_{ij} \quad \forall i \in V \tag{7.48}$$

$$f_j^k \geq f_i^k + L_j^k - U_j^k - 2C^a (1 - x_{ij}) \quad \forall k \in K \forall i \in V \tag{7.49}$$

$$0 \leq L_i^k \leq b^k \quad \forall i \in O_k \cup G_k \forall k \in K \tag{7.50}$$

$$0 \leq U_i^k \leq b^k \quad \forall i \in D_k \cup G_k \forall k \in K \tag{7.51}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V, \forall j \in V, E_{p(i)}^a = 0 \wedge E_{p(j)}^a = 0 \tag{7.52}$$

$$f_i^k \in \mathcal{R}^+ \quad \forall i \in V \forall k \in K \tag{7.53}$$

$$\lambda_S \in \{0, 1\} \quad \forall S \subseteq V \tag{7.54}$$

$$n_S \in \mathcal{Z}^+ \quad \forall S \subseteq V \tag{7.55}$$

A solution to the pricing problem for vessel class a will be a set of cycles as explained in section 7.4.3. In order to identify y_{rw}^a variables it is necessary to identify all subsets $S \subseteq V$ containing a cycle by the variables $\lambda_S = 1$ and identify the corresponding loading pattern $w \in W$ by the L_i^k, U_i^k variables for all $i \in S$. The obvious drawback of the pricing problem is the exponential set of constraints (7.41) -(7.42) and variables (7.54)-(7.55) required to model every subset with a cycle and assign an integer number of vessels to each cycle. For solving the pricing problem we propose to relax (7.41)-(7.43) with the constraint:

$$\sum_{(i,j) \in \delta(S)} x_{ij} \frac{d_{p(i)p(j)}}{W^a} \leq N_c$$

This relaxation will return a set of cycles, that may be non-simple and which individually may deploy a fractional number of vessels of class a . Post processing can identify each individual non-simple cycle and its corresponding cargo pattern giving us a set of master variables, which jointly has reduced cost. Once y_{rw}^a variables have been identified the reduced cost \hat{c}_{rw}^a of each individual cycle, is calculated to determine, whether it has positive reduced cost. Variables with positive reduced cost should be added to the master problem.

7.4.5 Solving the pricing problem

To date we are not aware of a practically usable algorithm for solving the pricing problem as it is defined in (7.39)-(7.55) to optimality. The pricing problem does not lend itself well to the resource constrained shortest path algorithms seen in the split delivery VRP or Pick-up-and-Delivery VRP, because the variables represent a cycle as opposed to a path. This poses two problems: The resources are cyclic and it is difficult to compute the reduced cost as the vessels are never empty. Secondly, the routes do not have a designated starting point which removes all precedence relations from the resources. It is possible to solve the pricing problem heuristically, perhaps in a two phased approach constructing a route and hereafter solving a multidimensional knapsack problem for the commodities based on the reduced cost. The solution for constructing cargo patterns in this manner must incorporate transshipments by defining a set of demands, that are either transshipped or transported directly. It is not unlikely that the pricing problem in its entirety can be solved using a MIP solver for smaller instances, as it is a variant of the selective Travelling Salesman Problem, where no subtour elimination constraints are imposed. However, it will undoubtedly not be efficient to solve the subproblems using a MIP solver in general due to the exponential set of constraints and variables to ensure integral deployment of vessels to cycles. It may prove computationally efficient to solve the pricing problem for a small subset of the set of nodes. Alternative relaxations of the pricing problem could be investigated in order to be able to obtain upper bounds for the problem, such as a tree-based relaxation.

7.5 Conclusions on modelling

The distinct models published on LSNDP reveal variants of the MCND with high degrees of freedom and a vast solution space of mathematical models. In particular partial demand satisfaction result in large integrality gaps. Introducing minimum deliveries of a subset of demands in the LSNDP would enable introduction of valid inequalities e.g. the cut-set inequalities known from MCND's. Constraints may also be imposed upon the cargo flow problem such as interbalancing constraints to account for empty repositioning and imposing transit time restrictions for individual commodities. This is an implicit constraint on the integer variables of the problem. This chapter has discussed approaches seeking to reduce the integrality gap by using models inspired by the VRP to circumvent the linear multicommodity flow problem. The model in Jepsen et al. (2011) and the model presented in this chapter reveal decomposition schemes with complicated pricing problems, that require extensions to known efficient methods or alternative solution methods. This leaves several open research questions in modelling and solving such mathematical models of the LSNDP to optimality. In this Chapter a model built upon the idea of Jepsen et al. (2011) has been presented as we believe this research direction to be a promising. Relaxations of the pricing problem may be used initially to produce bounds for the branching procedure. A very rough sketch of a solution method has been discussed, but further research into constructing good heuristics producing a cycle and/or a cargo pattern must be undertaken and an actual implementation is needed to evaluate the quality of the model.

Heuristic methods may be applied to the reference model presented in Chapter 2, but even developing heuristics is a complex and challenging task for the LSNDP as we have seen in Chapters 2 and 5.

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Part IV

Conclusion

Chapter 8

Conclusion

This thesis has highlighted various aspects of liner shipping network design seen in the context of mathematical modelling and optimization. Chapter 2 describes the domain of liner shipping network design problems from the perspective of a global carrier and relates the domain to a reference model of the core concepts of liner shipping network design. In conjunction with this a set of realistic benchmark instances have been created, which have been made publicly available to the research community to stimulate interest into this important transportation problem. A heuristic column generation algorithm is described and computational results are reported for small, medium and large instances of the benchmark suite. Best known solutions are provided as an offset for the benchmark suite.

The heuristic column generation technique provides good solutions for all instances, except a constructed case (the Mediterranean case). A challenge for heuristic methods is the performance related to evaluating very large scale and complex neighbourhoods and also the performance of evaluating a given move due to an underlying very large scale multicommodity flow problem. The performance issues makes it essential to make larger moves in the vast solution space of the LSNDP, and at the same time to make moves of very high quality. The heuristic column generator in Chapter 2 uses a MIP to make new routes for preselected clusters of ports. The MIP must be solved for a number of combination of vessel classes, number of vessels deployed and average speed of the services. The MIP continues to find improved solutions and determine services of high quality, but it converges slowly and each iteration is computationally expensive. In Chapter 5 we presented a matheuristic for the LSNDP, which aims to be computationally efficient in terms of the time to search a neighbourhood for the most promising move(s), and in terms of quickly evaluating a move. A *Delta column generator* is introduced, which uses the optimal solution of the previous multi commodity flow problem adjusted for the local changes introduced by a given move. The *Delta column generator* gives a performance increase in terms of execution times of a factor 2-10 on average. The execution time of the warm starts vary greatly depending on the centrality of the move in terms of the number of variables affected (i.e. the percentage of the warm started basis that can be reused). The primary neighborhood of the matheuristic is an integer program defined for each service estimating the value of inserting and removing ports in a service. This MIP neighbourhood is intended to have a linear growth in evaluation time in relation to the number of ports in the instance. A growth in the number of ports will typically mean a larger set of services, but searching a neighbourhood of a single service remains stable with growing instance size. The time to calculate the estimation functions needed to solve the MIP neighborhood is low as well as solution times that averages to less than a second for most instances. The matheuristic is combined with a local search framework attempting to adjust the total number of services, the vessel class deployment and the number of vessels deployed to each service. The MIP neighbourhoods are able to fine tune a given solution to the demands, but cannot change the initial composition of the services or the vessel class deployed. However, the construction heuristic provides a poor offset for the optimization and the local search must be effective in transforming the solution. The results of the matheuristic display a large deviation between the

best result found and the average of ten runs, but it is significantly faster than the heuristic column generator. The approach can be combined with more effective local search methods and possibly an improved construction heuristic. Experiments indicate that a more advanced method to generate high quality new services during the search would improve the search of the solution space. The results from the heuristic column generator in Chapter 2 cannot be compared to the solutions using the matheuristic due to different frequency requirements. The best solution for two cases found by the heuristic column generator are adjusted to meet strict weekly frequency as required by the matheuristic. Experiments using this best solution from the heuristic column generator as start solution shows improvements indicating that the matheuristic is able to produce good results given a good initial solution. Additionally, the matheuristic has been tested as a decision support tool, where a large network is given as input and a subset of the services are optimized using the matheuristic. These experiments indicate that the matheuristic may provide valuable decision support to network planners when incrementally adjusting their network to changes in the markets for cargo and vessels.

The thesis also explore tactical and operational problems, where the overall network is fixed to a greater extend. We have explored a tactical problem determining cargo flows considering the significant repositioning of empty containers, which are inherent to the liner shipping network due to trade imbalances. Traditionally the OR community has considered empty repositioning as a post optimization process once the cargo transportation in the network has been fixed, creating a given supply/demand of empty containers to be repositioned in the network.

In Chapter 3 a mathematical model has been presented for routing the most profitable cargo on a given network considering the cost for repositioning empty containers to points of demand. The mathematical model is a Multicommodity Flow Problem with interbalancing constraints. An arc flow model is presented and a path-flow decomposition is presented. The path flow model is solved using delayed column generation and computational results confirm that it is possible to evaluate a realistic sized network over a longer planning period including the flow of empty repositioning to evaluate network performance. Additionally, the solution may help determine, which commodities are preferred for transport in the network. This can provide valuable decision support to network planners, who may use the tool to evaluate changes to the network, changes in the demand or alternative pricing schemes to optimize upon an existing network.

The Cargo allocation with Empty Repositioning model can be extended to handle additional complexities such as a maximum transit time for each demand by adding a transit time constraint for each demand and calculate the paths for a demand with a cost and a transit time. The pricing Problem will become \mathcal{NP} -hard, but can be solved as a resource constrained shortest path problem for which quite effective dynamic programming algorithms are known. Cabotage rules may likewise be incorporated as a resource. This extension would be very valuable to the network planners as they would be able to incorporate important real life traits of a good liner shipping network design.

Another paramount optimization problem in liner shipping is disruption management as 70-80% of all round trip voyages experience delays in one or more ports. In Chapter 4 a novel model for recovering from a given disruption scenario has been presented. The model is able to combine different recovery techniques considering the cost increase including a penalty for misconnecting and delaying containers on board affected vessels. The model has been tested on four common real life scenarios from Maersk Line. The solutions identified by the model are compared to the realised recovery actions performed at Maersk Line and results reveal solutions that are tantamount or improved in terms of costs and misconnections. Future research may investigate a post optimization process of rerouting the disrupted flow on the residual capacity of the resulting network in order to accommodate efficient and easy transport for the misconnected and delayed containers, without altering the flow of the remaining network.

Identifying tactical and operational problems inherent to the current business process of incrementally adjusting and improving a given network design is possible given state-of-the-art optimization techniques and scales well to liner shipping network instances of realistic size. Providing decision support at this level to liner shipping network carriers appears to be a promising field for future research, where success of actual applications may spur industry interest into optimization

and decision support using mathematical modelling and optimization techniques.

The reference model in Chapter 2 is one mathematical interpretation of the liner shipping network domain seen from a global carrier, but the model has an obvious drawback as the set of variables is exponential and efficient methods to generate variables using delayed column generation is complicated by a set of rows specific to each variable. A model with the same structure is presented in Stålhane et al. (2012), where the complication is handled by obtaining a proof of optimality with an implicit representation of all variables in the problem.

An interesting line of research would be to pursue a similar proof for optimality of the reference model with a partial set of variables and a definition of the pricing problem combined with an optimal solution algorithm for the pricing problem. This would open for development of efficient exact solution methods for the reference model. Obtaining good bounds would help evaluate the solutions provided by heuristic methods such as the heuristic column generator and the matheuristic presented in this thesis. The reference model is a variant of the Multicommodity Capacitated Network Design problem (MCND), which is recognised in the community as a very hard optimization problem (Gendron et al., 1999). One challenge in MCNDs is the large integrality gap of the LP relaxation resulting in large Branch-and-Bound trees and poor bounds to effectively prune the tree. Chapter 6 presented an alternative mathematical model inspired by the development of Branch-and-Price-and-Cut algorithms using delayed column generation within the VRP community. This model combines routing with a cargo loading pattern in an attempt to close this integrality gap by implicitly representing the multicommodity flow problem of transporting cargo on the integer variables. The mathematical model reveals a very complex pricing problem of creating a longest cycle with cargo loads. Designing an efficient dynamic programming algorithms for this pricing problem is not trivial as dominance criterias are weak and the state space is huge. Relaxations of the pricing problem and primal heuristics of high quality is needed to make an effective Branch-and-Price-and-Cut algorithm for this model. The mathematical model is based on a novel trade based aggregation of the commodity set effectively reducing the number of demands. However, this aggregation also blocks an obvious relaxation of the pricing problem, where it is required to load the maximum amount of a commodity in order to reduce the state space. The maximum is the maximum of the total demand or the available capacity.

In Chapter 7 a model for the LSNDP with weekly frequency has been presented, inspired by the model in Chapter 6. The difference consists in a decomposition based on the set of vessel classes instead of the set of services and a minimum delivery of each demand is introduced enabling the use of cut-set inequalities for a subset of the demands. The decomposition results in a pricing problem for each vessel class returning a set of routing variables for each vessel class with a corresponding cargo pattern. The pricing problem introduces cargo loading patterns as seen in Desaulniers (2010) and Stålhane et al. (2012) to reduce symmetry of the problem, which should result in a more scalable pricing problem. However, exact solution methods for this pricing problem are equally complex and at the time of writing we do not know efficient methods to solve the pricing problem to optimality.

Again, an interesting line of research would be to find a good relaxation of the pricing problem and design efficient primal heuristics in order to obtain good bounds for the model using Branch-and-Price-and-Cut algorithms in a heuristic fashion. This was the offset for the pricing problem of the VRP problems back in the early days of designing column generation algorithms for VRP variants. The mathematical models are still considered promising in spite of their complexity and hopefully, future research will be able to close the gap by finding good relaxations and eventually efficient methods to solve the pricing problems to optimality. Luckily research within PDVRP with split deliveries are progressing and the VRP community is starting to address more complex variants with multiple commodities and cross dockings, which may help solve some of the challenges present in the pricing problem of constructing routes with cargo patterns for LSNDP.

In conclusion the development of mathematical models for the LSNDP is ongoing and it is a challenge to construct a mathematical model capturing the core concepts in a liner shipping network, while obtaining a tight formulation suitable for designing efficient MIP and decomposition algorithms. The thesis explores three models that closely reflects the core of a real life operation of a global carrier such as weekly frequencies, extensive use of transshipments and non-simple (butterfly)

cyclic routes. In time additional complexity of models is required to reflect level of service by imposing a maximal transit time and designing a schedule, where speed optimization is considered for each individual voyage of a vessel. Empty repositioning could possibly be incorporated without adding increased complexity to current models. Furthermore, current models are deterministic and demand fluctuates with freight rates and market conditions, which is not reflected in current models. The field is young and we need to master the core model, before we start adding complexity to the models such as stochastic demands, variable speed and level of service.

Part V

Appendix

Appendix B

A greedy construction heuristic for the Liner Service Network Design Problem

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Abstract The Liner Service Network Design Problem (LSN-DP) is the problem of constructing a set of routes for a heterogeneous vessel fleet of a global liner shipping operator. Routes in the liner shipping context are non-simple, cyclic routes constructed for a specific vessel type. The problem is challenging due to the size of a global liner shipping operation and due to the hub-and-spoke network design, where a high percentage of the total cargo is transshipped. We present the first construction heuristic for large scale instances of the LSN-DP. The heuristic is able to find a solution for a real life case with 234 unique ports and 14000 demands in 33 seconds.

Literature overview: A MIP model of the LSN-DP consist of a highly unconstrained routing problem subject to a large degree of symmetry and a multicommodity flow problem dominating the constraint set and accountable for a large fraction of the cost. Previous work on liner service network design may be found in Rana and Vickson (1991), Reinhardt and Kallehauge (2007), Agarwal and Ergun (2008) and Alvarez (2009). The models are distinct with regards to transshipments and the vessel fleet. In the early paper of Rana and Vickson (1991) transshipments are not supported, whereas they are supported in Reinhardt and Kallehauge (2007), Agarwal and Ergun (2008) and Alvarez (2009). Transshipment costs are accounted for in the objective function of Reinhardt and Kallehauge (2007), Alvarez (2009) as opposed to Agarwal and Ergun (2008). The characteristics of the fleet differ as to whether it is heterogeneous for every vessel Rana and Vickson (1991), Reinhardt and Kallehauge (2007) or consist of a heterogeneous fleet of homogeneous vessel classes Agarwal and Ergun (2008), Alvarez (2009). Non-linear capacity constraints are found in Rana and Vickson (1991), Reinhardt and Kallehauge (2007) assuming that a vessel may complete its route an integral number of times during the planning horizon. Integrality is not imposed by Alvarez (2009). The weekly frequency constraint is introduced by Agarwal and Ergun (2008) assuming a number of homogeneous vessels assigned to each service to offer a weekly visit to each port en route with the capacity of the vessel class in question. Optimal results for smaller instances are presented by Reinhardt and Kallehauge (2007) and Alvarez (2009). The approaches

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of Rana and Vickson (1991), Reinhardt and Kallehauge (2007) and Agarwal and Ergun (2008) have focused on solving the liner service network design problem with traditional decomposition and integer programming methods and fail to produce results for realistic network sizes of a global liner shipping operator anno 2009. This is addressed by Alvarez (2009) benchmarking a tabu search approach presenting results within 3-5% of the optimal solution for up to 7 ports. The multicommodity flow problem is solved in each iteration, which is reported by Alvarez (2009) to become computationally expensive already for the 7 port instance. A case study of 120 ports in Alvarez (2009) show that a heuristic approach may scale to large instances but no execution time is reported and the quality of the solution is hard to evaluate. It is reported to visit important ports infrequently. A global network connects several hundred ports worldwide and the corresponding forecasted cargo demand comprises a commodity set of 4 orders of magnitude. Methods based on relaxation of the proposed models or simply evaluating the objective function during a search is not computationally efficient for large scale problems.

Heuristic approach: A solution to the LSN-DP is a set of routes covering the ports serviced by the shipping operator and transporting the forecasted cargo demand. Viewed as a graph partitioning problem the solution is a set of strongly connected components with a high degree of interconnection. The construction heuristic is based on the Multiple Quadratic Knapsack Problem (MQKP) and relies on a graph of the current schedule, which is divided into a set of dense subgraphs related by demand, expected transshipment flow and geographical proximity. A solution found by the construction heuristic is expected to be feasible and realistic, but the quality of the solution cannot be guaranteed as the heuristic cannot account for the flow problem and the transshipment cost. In MQKP a set of mutually exclusive items $i \in V$ are placed in \mathcal{R} knapsacks with different weight bounds C_r . The objective is to maximise the profit of the knapsacks defined by the profit matrix P .

$$\text{maximize}(\text{MQKP}) = \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} p_{ij} x_i^r x_j^r + \sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{V}} p_j x_j^r \quad (\text{B.1})$$

subject to:

$$\sum_{i \in \mathcal{V}} w_i x_i^r \leq C_r \quad \forall r \in \mathcal{R} \quad (\text{B.2})$$

$$\sum_{r \in \mathcal{R}} x_i^r \leq 1 \quad \forall i \in \mathcal{V} \quad (\text{B.3})$$

$$x_i^r \in \{0, 1\} \quad \forall i \in \mathcal{V} \quad (\text{B.4})$$

The variables x_i^r indicate whether item i is included in the r 'th knapsack. The knapsack constraint (B.2) makes the total item weight obey the bound C_r and constraints (B.3) ensure that items are mutually exclusive to the knapsacks. When the MQKP is applied to the LSN-DP the knapsack items $i \in V$ are the accumulated port visits of each port $t \in T$ and the knapsacks $r \in R$ represent services, which are a specific vessel class visiting a sequence of ports. Let A be the set of vessel classes and let N_a denote the number of available vessels of class $a \in A$ in the fleet. Let C_a denote the capacity in TEU of a single vessel of class $a \in A$. The number of services/knapsacks is dependent on the expected rotation time of a service $\sigma(C_a)$. $\sigma(C_a)$ depends on vessel capacity as large vessels are typically assigned to cross regional services and small vessels are assigned to regional services. The number of knapsacks for the LSN-DP is hence $|R| = \sum_{a \in A} |R_a| = \sum_{a \in A} \lceil \frac{N_a}{\sigma(C_a)} \rceil$. The profit matrix \mathcal{P} defines each entry $p_{ij} = f(l_{ij}, d_{ij}, h_{ij})$ where l_{ij} is the sailing distance in nautical miles and d_{ij} is the demand between ports $i, j \in V$. h_{ij} is the potential hub flow between port $i \in V$ and a hub port $j \in H \subset V$, where $H \subset V$ are ports with a small percentage of demand compared to the terminal capacity. A port $t \in T$ may be visited multiple times m_t by multiple services according to the capacity and schedule requirements of a port. Let M be a vector of size $|T|$ containing the number of weekly visits to each port $t \in T$. In the MQKP port $t \in T$ is duplicated m_t times for the knapsack items $i \in V$, $V = \{T \times M\}$ to represent the current schedule of ports. It is important to observe the weekly frequency constraint of the original problem in order to obtain a feasible solution to the LSN-DP using the construction heuristic. To

ensure feasibility, each knapsack $r \in R$ is required to provide a Hamiltonian cycle of the items in knapsack r . The length of the cycle cannot exceed the mileage coverable by the vessels assigned to knapsack r . Edge variables y_{ij}^r and enumeration variables u_i^r are introduced in the MQKP to order the ports in each knapsack into a simple, cyclic route constrained by $\sigma(C_a)$. t_{ij}^a express the sailing time between ports i and j and t_i^a is the expected time spent in port i for vessel type a .

$$\text{maximize}(MQKP) = \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} p_{ij} x_i^r x_j^r + \sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{V}} p_j x_j^r \quad (\text{B.5})$$

$$\text{subject to: } x_i^r x_j^r \geq y_{ij}^r \quad \forall i \in \mathcal{V}, j \in \mathcal{V}, r \in \mathcal{R} \quad (\text{B.6})$$

$$\sum_{j \in \mathcal{V}} y_{ij}^r - \sum_{j \in \mathcal{V}} y_{ji}^r = 0 \quad \forall i \in \mathcal{V}, r \in \mathcal{R} \quad (\text{B.7})$$

$$\sum_{j \in \mathcal{V}} y_{ij}^r \leq 1 \quad \forall i \in \mathcal{V}, r \in \mathcal{R} \quad (\text{B.8})$$

$$u_i^r - u_j^r + y_{ij}^r \sum_{i \in \mathcal{V}} x_i^r \leq \sum_{i \in \mathcal{V}} x_i^r - 1 \quad \forall i \in \mathcal{V}, j \in \mathcal{V}, r \in \mathcal{R} \quad (\text{B.9})$$

$$\sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} y_{ij}^r (t_{ij} + t_i) \leq \sigma(C_a) \quad \forall r \in \mathcal{R}_a, a \in \mathcal{A} \quad (\text{B.10})$$

$$\sum_{r \in \mathcal{R}} x_i^r = 1 \quad \forall i \in \mathcal{V} \quad (\text{B.11})$$

$$x_i^r \in \{0, 1\} \quad \forall i \in \mathcal{V}, r \in \mathcal{R} \quad (\text{B.12})$$

$$y_{ij}^r \in \{0, 1\} \quad \forall i \in \mathcal{V}, j \in \mathcal{V}, r \in \mathcal{R} \quad (\text{B.13})$$

$$u_i^r \in \mathbb{Z}^+ \quad \forall i \in \mathcal{V}, r \in \mathcal{R} \quad (\text{B.14})$$

Constraints (B.6) ensure that an edge variable can only be activated if both endpoints of the arc are included in the knapsack. Constraints (B.7) ensure a cyclic route. Constraints (B.8) ensure that the cyclic route is simple and constraints (B.9) that the route is connected. Constraints (B.10) are the weekly frequency constraint ensuring that the simple, cyclic route has a voyage duration less than the expected rotation time.

The MQKP is solved using a greedy heuristic, where the knapsacks apply the football teaming principle taking turns at picking the best remaining item $\max \Delta f(l_{ij}, d_{ij}, h_{ij}), i \in r, j \in \bar{V}$ where \bar{V} are the unassigned items.

In a hub-and-spoke network design large vessels are deployed on deep sea services to achieve economies of scale Shintani et al. (2007), while smaller vessels are deployed between spoke and hub. The algorithm is multilayered to reflect the hub-and-spoke network design of major liner shipping operators. The function $f(l_{ij}, d_{ij}, h_{ij})$ is adapted to each layer of the network and the ports are correspondingly assigned to the layers according to their capacity requirements.

Computational results: The computational results are based on a real life case from Maersk Line with 234 unique ports and 14000 demands. The MQKP is able to find a solution in 33 seconds for the entire network with some ports unplaced. The solutions have been evaluated by optimization managers at Maersk Line regarding them as realistic with some modifications. Current work is on implementing layer specific seeding to improve the number of unplaced ports. We evaluate the actual flow and the network cost of the solution. We believe that meta heuristic approaches are needed to optimize liner service networks of global shipping operators and are working on specializing the Adaptive Large Neighbourhood Search Pisinger and Ropke (2007) VRP framework towards the context of transshipments and cyclic routes. The first step is the construction heuristic for the LSN-DP presented here. The ALNS is to search for an improved solution according to a more sophisticated objective function and cargo allocation detection.

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